

**Mathematics
Practice Test Booklet
for the PSSA**

**Grade 11
Question 73**



A container must hold 400 cubic inches of sand. Accurately sketch and label the dimensions of a rectangular prism, sphere, and cylinder which will each hold the 400 cubic inches of sand with the least amount of wasted space. Show the calculations that verify that each container will hold 400 cubic inches.

Show your work and explain the steps you used to justify your answer. Do all work for this problem in the box below. Remember you must show all the steps you used to solve the problem even if you have used a calculator. To receive the highest score, all calculations steps must be shown and explained in writing. Numeric answers must always be labeled.

Problem Solution:

Rectangular Prism: An infinite number of solutions are possible. Some sample solutions are

$$V = lwh = 400$$

$$V = (10)(10)(4)$$

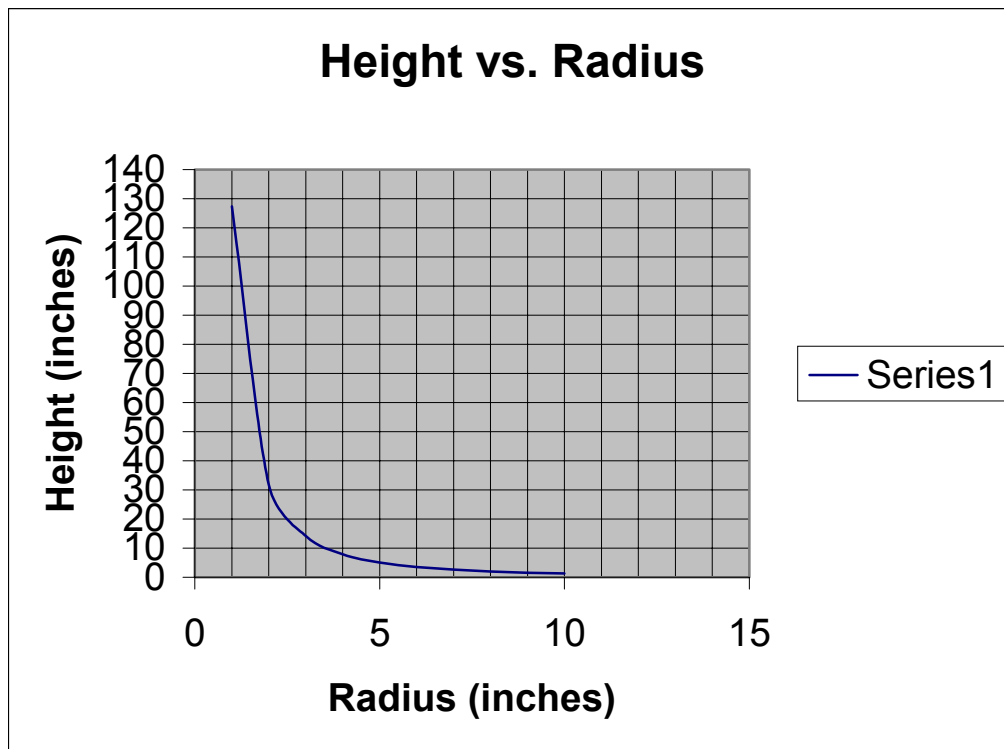
$$V = (10)(8)(5)$$

$$V = (20)(4)(5)$$

$$V = (40)(10)(1)$$

$$V = (20)(10)(2)$$

Cylinder: An infinite number of solutions are possible. Some sample solutions are $V = \pi r^2 h$



$$400 = \frac{4}{3} \pi r^3$$

$$\frac{3(400)}{4\pi} = r^3$$

$$95.493 = r^3$$

$$4.571 = r$$

Sphere: For a 5 or 4, $4.571 \leq r \leq 4.572$

For a 3, $4.571 \leq r \leq 4.601$

5 ADVANCED UNDERSTANDING

(Correct solution for all three parts with all work shown explained resulting in a volume $400 \leq V < 400.5 \text{ in}^3$ on all parts [see solution sheet]. Sketches are correct and labeled. "Why" statements are given. Units are required on all parts.)

4 SATISFACTORY UNDERSTANDING

(Correct method resulting in a volume $400 \leq V < 400.5 \text{ in}^3$ on all parts. Units are required on at least one part.)

4A Correct solution with all work shown and some explanation

4B Correct solution, sketches may be slightly flawed. All work shown or explained.

3 ALMOST UNDERSTANDING

(Correct method resulting in volume $400 \leq V \leq 408 \text{ in}^3$. Units are required on at least one part.)

3A Correct solution for all three parts with some work and some explanation

3B Incorrect solution due to one calculation or rounding error

3C Correct solution with work and explanation, sketches omitted

3D 2 out of 3 correct solutions with sketches and all calculations shown and explained (no errors)

3E Incorrect solution due to one calculation error and no verification of volume

2 PARTIAL UNDERSTANDING

(At the 2 level, sketches are not required unless stated otherwise.)

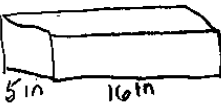
- 2A** Correct solution with minimal work or some explanation
- 2B** Incorrect solution due to multiple calculations errors (including approximations that cause $V > 408 \text{ in}^3$)
- 2C** Incorrect answer due to a conceptual error such as use of a pyramid or other prism or volume less than 400 in^3 on at least one part
- 2D** Finding 2 out of 3 solutions and values correct within rounding, some work and explanation, may contain a calculation error
- 2E** Find 1 out of 3 correct solutions with all work shown, explanation and sketch

1 MINIMAL UNDERSTANDING

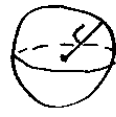
- 1A** Correct solution with no work or explanation
- 1B** Sketches of the 3 correct figures only
- 1C** 1 out of 3 correct answers (no explanation and no work but formulas)
- 1D** 1 correct answer and an attempt at another part ($r = 4.57 \text{ in.}$ for sphere OK)

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$L = 16 \text{ in}$ $V = LWH$
 $5 \text{ in } W = 5 \text{ in } V = 5 \cdot 5 \cdot 16$
 $H = 5 \text{ in } V = 400 \text{ in}^3$



$V = \frac{4}{3}\pi r^3$
 $r = 4.572 \text{ in}$ $V = \frac{4}{3}\pi(4.572)$
 $V = 400.32 \text{ in}^3$



$r = 7.98 \text{ in}$ $V = \pi r^2 h$
 $h = 2 \text{ in}$ $V = \pi(63.6)(2)$
 $V = 400.116 \text{ in}^3$

To find the L, W, and H of the rectangular prism I used the guess and check method until I came across the correct answer. I multiplied 3 #'s together to see how close to 400 they were. With the sphere I basically reversed the formula. I divided 400 by $\frac{4}{3}$ which equals 300. Then I divided that by 3.14 (π) and got 95.541. I took the cube root of that to get 4.572 as my radius. The cylinder I also reversed the problem. The only thing I changed there was I had to guess once I divided by π . All the answers I got are very close to 400 all would all work.

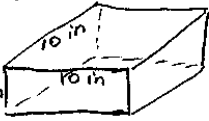
5 – Holistic explanation. Discusses when guessed (forced to) and when didn't. All answers within acceptable ranges. Units listed.

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Prism
 $400 = LWH = I$ made the required volume equal to the formula for volume of a prism
 $400 = 10(WH)$ I pick a random logical # for the length
 $400 = 10(10)(H)$ I pick a random "10" "10" "10" width
 $400 = (10)(10)(H)$ I divided 400 by 100 (the L and W) and got height to be 4

$LWH = U$
 $(10)(10)(4) = U$
 $(100)(4) = U$
 $400 = U$




The demensions of the prism which would hold 400 cubic inches of sand would be 10 in x 10 in x 4 in

Sphere
 $400 = \frac{4}{3}\pi r^3$ I made the required volume equal to the formula for volume of a sphere.
 $3(400) = (\frac{4}{3}\pi r^3) \cdot \frac{3}{4}$ I multiplied by $\frac{3}{4}$ to start solving for r the only variable
 $\frac{300}{\pi} = \pi r^3$ I divided by π on both sides to get the variable alone
 $\sqrt[3]{95.493} \approx r$ I took the $\sqrt[3]{}$ of 95.5 to get what r must equal if there must be 400 cubic inches of area in this sphere.
 $4.571 \text{ in.} \approx r$ The radius of a sphere that holds 400 cubic inches of sand must be about 4.571 inches.

$(\pi r^3) r \approx 4.571 \text{ inches}$ $\frac{4}{3}\pi (4.571)^3 = U$ $\frac{4}{3}\pi (95.507) = U$ $127.343\pi = U$ $U = 400.06\pi$

Cylinder
 $\pi r^2 h = 400$ = I made the required volume equal to the formula for the volume of the cylinder
 $r^2 h = 127.324$ I divided by π to have the variables stand alone
 $r^2 = 12.733$ I made h equal to ten (I picked r and then divided by 14).
 $r = 3.57$ I took the square root of 12.733 and r and this makes $r = 3.57$ inches



$\pi r^2 h = U$ $40.039(10) = U$
 $\pi (3.57)^2 (10) = U$ $400.4 \text{ in} = U$
 $\pi (12.74)(10) = U$

5 – All answers within acceptable range. Step-by-step explanation of procedure.

A container must hold 400 cubic inches of sand. Accurately sketch and label the dimensions of a rectangular prism, sphere, and cylinder which will each hold the 400 cubic inches of sand. The sand should fill each container as much as possible with the least amount of wasted space. Show the calculations of the volume of each container to verify their accuracy to 400 cubic inches.

Show your work and explain the steps you used to justify your answers. Do all work for this problem in the shaded region below. Remember you must show all the steps you used to solve the problem even if you have used a calculator. To receive the highest score, all calculation steps must be shown and verbally explained. Numerical answers must always be labeled.

rectangular prism

Volume = (length)(width)(height)

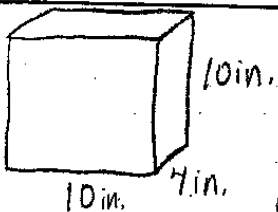
substitute 400 in³ for volume.

$400 = LWH$

pick out any 3 numbers that would multiply out to equal 400.

$400 = (10)(4)(10)$

$400 \text{ in}^3 = (10 \text{ in})(4 \text{ in})(10 \text{ in})$



cylinder

Volume = $\pi r^2 h$


substitute 400 in³ for volume

$400 = \pi r^2 h$

guess and check numbers for r and h. keep refining #'s until the outcome is 400 or more. try to get numbers that come as close to 400 as possible without going under it.

$r = 4$
 $h = 7.96$

$V = \pi(4^2)(7.96)$
 $V \approx 400.11$



sphere

Volume = $\frac{4}{3} \pi r^3$

substitute 400 in³ for volume

$400 = \frac{4}{3} \pi r^3$

solve for radius.

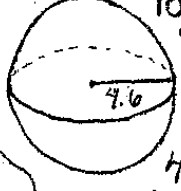
$\frac{3}{4} \cdot 400 = \frac{4}{3} \pi r^3 \cdot \frac{3}{4}$ multiply by reciprocal.

$300 = \pi r^3$ divide each side by π .

$\sqrt[3]{\frac{300}{\pi}} = r^3$ Find the cube root of each side to get r alone.

$r = \sqrt[3]{\frac{300}{\pi}}$ since this number is irrational, you have to approximate however you most round up so there is at a minimum 400 cubic in. of volume.

$r \approx 4.6$



4A – “Why” step listed. Some checks shown (not for sphere). No units on cylinder or sphere. (Guesses not shown).

A container must hold 400 cubic inches of sand. Accurately sketch and label the dimensions of a rectangular prism, sphere, and cylinder which will each hold the 400 cubic inches of sand. The sand should fill each container as much as possible with the least amount of wasted space. Show the calculations of the volume of each container to verify their accuracy to 400 cubic inches.

Show your work and explain the steps you used to justify your answers. Do all work for this problem in the shaded region below. Remember you must show all the steps you used to solve the problem even if you have used a calculator. To receive the highest score, all calculation steps must be shown and verbally explained. Numerical answers must always be labeled.

The volume of a rectangular prism can be found $V = LWh$. The volume specified is 400 cubic inches. I use the length of 50 inches. I now have the equation $400 \text{ in}^3 = 50 \text{ in} (Lh)$. 400 divided by 50 equals 80. The product of the width and height must equal 80. I will use the height of 10 and the width of 8. The length times width times height equals 400 in³.

To find the volume of a sphere, I will use the formula $V = \frac{4}{3} \pi r^3$. The volume is 400 in³. So $400 \text{ in}^3 = \frac{4}{3} \pi r^3$. 400 divided by $\frac{4}{3}$ equals 300. This means πr^3 must equal 300. Since π is approximately 3, I will substitute that for π . $300 = 3r^3$, $100 = r^3$. Since $100 = r^3$, I take the cube root of both sides and find that r is approximately 4.64.

The equation used to find the volume of a cylinder is $V = \pi r^2 h$. The volume is 400 in³. I will say that π is approximately 3. 400 divided by 3 equals $133\frac{1}{3}$. I will use the height of 15 in. This means I now have the equation $133\frac{1}{3} \text{ in}^3 = 15r^2$. I divide $133\frac{1}{3}$ by 15 and get approximately 9. This means I now have $9 = r^2$. I square root both sides and find that $r = 3$ in.

$V = LWh$
 $400 = 50Wh$
 $80 = Wh$
 $80 = 8(10)$
 $80 = 80$
 $L = 50 \text{ in}$
 $W = 8 \text{ in}$
 $h = 10 \text{ in}$

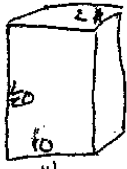
$V = \frac{4}{3} \pi r^3$
 $400 = \frac{4}{3} \pi r^3$
 $300 = \pi r^3$
 $100 = r^3$
 $4.64 \approx r$

$V = \pi r^2 h$
 $400 = \pi r^2 h$
 $133\frac{1}{3} = 15r^2$
 $9 = r^2$
 $3 = r$


2B – Volume of prism is 4000 (calculation error). Approximated π as 3 (not acceptable). $V = 418 \text{ in}^3$ too big sphere, $V = 424 \text{ in}^3$ too big cylinder.

A container must hold 400 cubic inches of sand. Accurately sketch and label the dimensions of a rectangular prism, sphere, and cylinder which will each hold the 400-cubic inches of sand. The sand should fill each container as much as possible with the least amount of wasted space. Show the calculations of the volume of each container to verify their accuracy to 400 cubic inches.

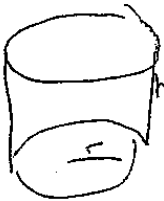
Show your work and explain the steps you used to justify your answers. Do all work for this problem in the shaded region below. Remember you must show all the steps you used to solve the problem even if you have used a calculator. To receive the highest score, all calculation steps must be shown and verbally explained. Numerical answers must always be labeled.




$V = LWh$
 $400 \text{ in}^3 = LWh$
 $L = 20 \text{ in}$
 $W = 10 \text{ in}$
 $H = 2 \text{ in}$
 $2 \cdot 10 \cdot 20$



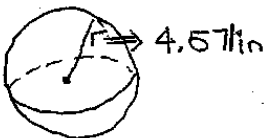
$V = \frac{4}{3} \pi r^3$
 $400 \text{ in}^3 = \frac{4}{3} \pi r^3$
 $400 \text{ in}^3 = \left(\frac{4}{3}\right) \left(\frac{\pi}{3}\right) (4.571)^3$



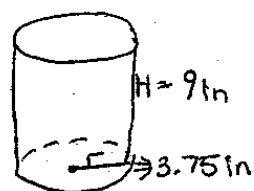
$V = \pi r^2 h$
 $400 \text{ in}^3 = (3.14)(r^2)(h)$
 $127 = (r^2)(h)$
 $\left(\frac{4^2}{8}\right)$
 $(3.5^2)(10) = 122.5$
 $(3.75^2)(9) = 126$



$W = 10 \text{ in}$, $L = 20 \text{ in}$, $H = 2 \text{ in}$



$r = 4.571 \text{ in}$




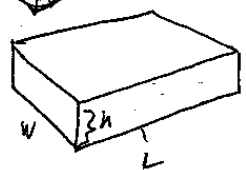
$H = 9 \text{ in}$, $r = 3.75 \text{ in}$


2C – Undershot volume of cylinder. No explanation, but all work shown.


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1.) 

1.)  $= 400 = LWh$
 $133.3 = S$

2.)  $= \frac{4}{3} \pi r^3$
 $400 = \frac{4}{3} \pi r^3$
 $300 = 4 \pi r^3$
 $75 = \pi r^3 \Rightarrow \sqrt[3]{235.493} = r^3 = 4.7 = r$

3.)  $400 = \pi r^2 h$
 $127.324 = r^2 h$

It would depend on the ~~for~~ purpose of the container, because spheres and cylinders leave empty ~~of~~ gaps in them when stacked, a box or prism doesn't, but a prism takes up the most external surface area for volume.

2C – Correct formulas. Correct sketches. Rounding error for Γ sphere. (4.7 not 4.57).

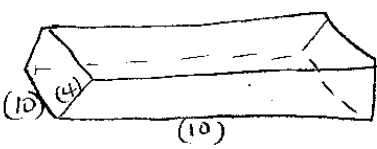
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$$V = Lwh$$

$$400 = Lwh$$

$$(4)(10)(10)$$



I calculated 400cm^3 volume as the volume of the prism + found logistical numbers to fit the formula.

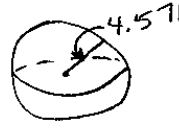
$$V = \frac{4}{3}\pi r^3$$

$$\frac{400}{\frac{4}{3}} = \frac{(\frac{4}{3})(\pi)r^3}{\frac{4}{3}}$$

$$\frac{300}{\pi} = \frac{\pi r^3}{\pi}$$

$$\sqrt[3]{95.49296586} \approx \sqrt[3]{r^3}$$

$$4.571 \approx r$$



I again calculated 400cm^3 as the volume of the cube and I solved for the radius.

~~$$V = \pi r^2 h$$~~


$$V = \pi r^2 h$$

$$\sqrt{400} = \pi \sqrt{r^2} h$$

$$\frac{20}{\pi} = \pi r h$$

$$6.366197724 = rh$$

$$(2)(\dots)$$



The volume was calculated to 400cm^3 . Then the radius + height were determined by logistics.

2D – No units with answers. Incorrect algebra for cylinder (conceptual error) and no check. ($V=40\text{ in}^3$ for cylinder).

A container must hold 400 cubic inches of sand. Accurately sketch and label the dimensions of a rectangular prism, sphere, and cylinder which will each hold the 400 cubic inches of sand with the least amount of wasted space. Show the calculations that verify that each container will hold 400 cubic inches.

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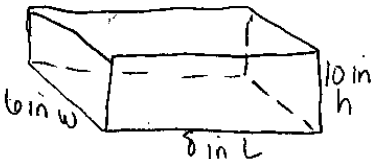
Rectangular prism:

$$V = lwh \quad A = 2lw + 2Lh + 2wh$$

$$V = 480 \quad A = 2(48) + 2(80) + 2(60)$$

$$= 376 \text{ in}^3$$

$l = 8$
 $w = 6$
 $h = 10$



If the length of the rectangular prism is 8 in, the width is 6 in, and the height is 10 in, the prism would hold 376 in³ of sand.


sphere:

$$V = \frac{4}{3}\pi r^3 \quad A = 4\pi r^2$$

$$V = 126.65 \quad A = 4(3.14)(30.25)$$

$$= 379.94 \text{ in}^3$$

$r = 5.5$



If the radius of the sphere is 5.5, the sphere would hold 379.94 in³ of sand.

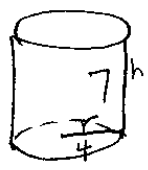
cylinder:

$$V = \pi r^2 h \quad A = 2\pi r^2 + 2\pi rh$$

$$V = 351.68 \quad A = 2\pi(16) + 2\pi(4)(7)$$

$$= 376.80 \text{ in}^3$$

$h = 7$
 $r = 4$



If the height is 7 and the radius is 4, the cylinder would hold 376.8 cubic in. of sand.

1B – Surface area (prism, sphere, cylinder). Correct formulas listed for both volume and surface area. Sketches correct.

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Hand-drawn sketches and calculations for three containers:

- Rectangular Prism:** A 3D sketch of a rectangular prism with dimensions labeled as 20 (width), 10 (depth), and 20 (height). The height is also labeled as "equals 2".
- Cylinder:** A simple sketch of a cylinder with a label "20 inches" next to it.
- Sphere:** A sketch of a sphere with a radius line drawn from the center to the surface, labeled "r = 200 inches".

Handwritten calculations:

$$V = LWh \quad 400 = Lwh$$

$$(2)(10)$$

$$20h = 400$$

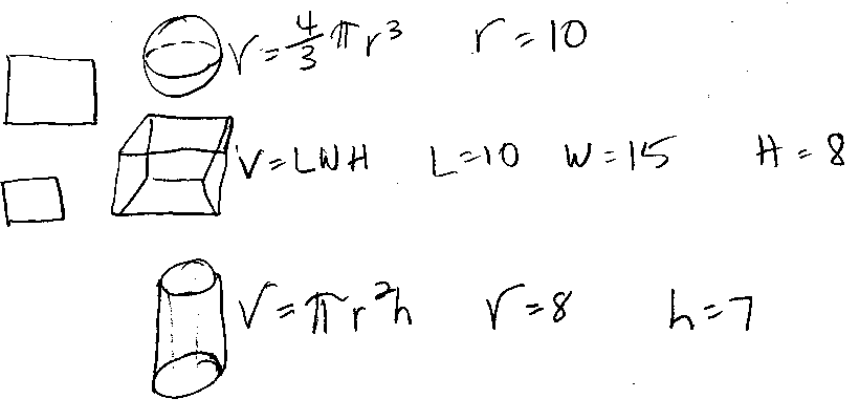
$$h = 20$$

$$400 = \frac{4}{3}\pi r^3$$

1B – Correct sketches only.

A container must hold 400 cubic inches of sand. Accurately sketch and label the dimensions of a rectangular prism, sphere, and cylinder which will each hold the 400 cubic inches of sand with the least amount of wasted space. Show the calculations that verify that each container will hold 400 cubic inches.

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$V = \frac{4}{3}\pi r^3$ $r = 10$

$V = LWH$ $L = 10$ $W = 15$ $H = 8$

$V = \pi r^2 h$ $r = 8$ $h = 7$

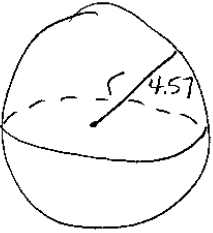
I took the 3 figures and found their measurements.

1D – Correct sketches. Only correct=prism, but no units, no explanations, no work.

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Show your work and explain the steps you used to justify your answer. Do all work for this problem in the box below. Remember you must show all the steps you used to solve the problem even if you have used a calculator. To receive the highest score, all calculations steps must be shown and explained in writing. Numeric answers must always be labeled.

sphere



$$V = \frac{4}{3} \pi r^3$$

$$(400 = \frac{4}{3} \pi r^3) \frac{3}{4}$$


$$\frac{1200}{4} = \pi r^3$$

$$\frac{300}{\pi} = \frac{\pi r^3}{\pi}$$

$$r = \sqrt[3]{\frac{300}{\pi}}$$

$$r = 4.57$$

cylinder



$$V = \pi r^2 h$$

$$400 = \pi r^2 h$$

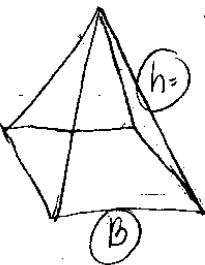
$$\frac{400}{\pi} = r^2 h$$

$$r = \sqrt{\frac{400}{\pi h}}$$

$$r = \frac{20}{\sqrt{\pi h}} \cdot \frac{\sqrt{\pi h}}{\sqrt{\pi h}} = \frac{20\sqrt{\pi h}}{\pi h}$$

$$h = \frac{400}{\pi r^2}$$

rectangular prism



$$V = \frac{1}{3} Bh$$

$$400 = \frac{1}{3} Bh$$

$$B = \frac{1200}{h}$$

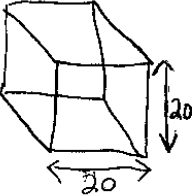
$$h = \frac{1200}{B}$$

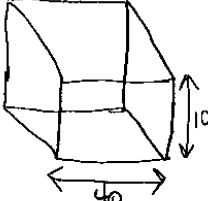
1D – Undershot volume of sphere. No final answer for cylinders. Conceptual error for rectangular prism.

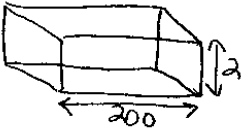
A container must hold 400 cubic inches of sand. Accurately sketch and label the dimensions of a rectangular prism, sphere, and cylinder which will each hold the 400 cubic inches of sand with the least amount of wasted space. Show the calculations that verify that each container will hold 400 cubic inches.

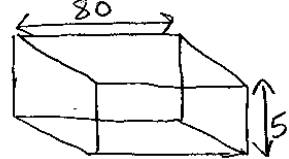
Show your work and explain the steps you used to justify your answer. Do all work for this problem in the box below. Remember you must show all the steps you used to solve the problem even if you have used a calculator. To receive the highest score, all calculations steps must be shown and explained in writing. Numeric answers must always be labeled.

$A = Bh$

①  the base times the height must equal 400. There are several possibilities. Here are a few.

② 

③ 

④ 

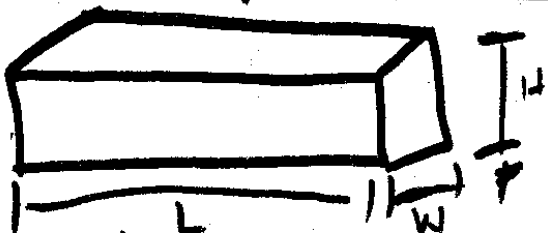
The formula to find area of a sphere and cylinder is necessary for the last two parts of the problem. I do not know these formulas.

0 – Incomplete solution (one figure, 2 dimensions for prism).

A container must hold 400 cubic inches of sand. Accurately sketch and label the dimensions of a rectangular prism, sphere, and cylinder which will each hold the 400 cubic inches of sand. The sand should fill each container as much as possible with the least amount of wasted space. Show the calculations of the volume of each container to verify their accuracy to 400 cubic inches.

Show your work and explain the steps you used to justify your answers. Do all work for this problem in the shaded region below. Remember you must show all the steps you used to solve the problem even if you have used a calculator. To receive the highest score, all calculation steps must be shown and verbally explained. Numerical answers must always be labeled.

RECT. PRISM



$2LW + 2LH + 2WH$
 400 in^3

$2LW + 2LH + 2WH = 400 \text{ in}^3$
 $2(LW + LH + WH) = 400 \text{ in}^3$
 $\frac{2}{2}$

$LW + LH + WH = \sqrt[2]{200 \text{ in}^3}$

0 – Surface area for only rectangular prism. Incorrect algebra indicated. No correct solutions.