Mathematics
Practice Test Booklet
for the PSSA

Grade 11
Question 50

The height, in feet, of a missile is given by **H** = $1600t - 16t^2$ and **t** is the time in seconds.

A) What is the maximum height reached by the missile?
B) What is the time it reaches the maximum height
C) What is the time when the missile hits the ground

For full credit, you **must** do the following:

1. show OR describe each step of your work, even if you did it in your head ("mental math") or used a calculator,

AND

2. write an explanation stating the mathematical reason(s) **why** you chose each of your steps.

Problem Solution:

Algebraic Method: set equation equal to zero to find factors

$1600t-16t^2=0$
$16t(100-t)=0$
$16t=0$ $\quad$ $100-t=0$
$t=0$ $\quad$ $100=t$

t=0 and t=100 represent when the height is 0. t=0 is before the missile is fired. t=100 is when it hits the ground. This answers part C.

t=100 sec. is the answer to part C

Since a parabola has symmetry the maximum point occurs on the axis of symmetry which is halfway between 0 and 100 (the roots) or can be found using $\frac{-b}{2a}$ to get the *x*-value of 50. So the answer to part B is 50 sec.
To find the maximum height substitute t=50 into the height equation.
$H=1600(50)-16(50)^2=80{,}000-40{,}000=40{,}000$ ft. The answer to part A is 40,000 ft.

Calculus Method: $H=1600t-16t^2$

Maximum occurs at critical values which are the roots of the first derivative.

$H^1=1600-32t$
$1600-32t=0$
$1600=32t$
50 sec=t

$H=0=1600t-16t^2$
$16t(100-t)=0$
$t=0$ $\quad$ $t=100$ $\quad$ [0, 100] domain

Hits the ground when t=100 sec because position returns to zero

Test for maximum value using intervals or using the second derivative test

(0, 50) (50, 100)
40 60
+ -
changes from increasing to decreasing a ∴ maximum

H^{11}=-32
Second derivative is negative therefore 50 is a maximum point.

H(50)=1600(50)-16$(50)^2$=40,000 ft. is the maximum height

Deductive Method:

Find zeros by factoring

$H=1600t-16t^2=0$
$16t(100-t)=0$
$t=0 \quad t=100$

t=0 sec is when the missile lifts off

t=100 sec is when the missile hits the ground

Since a parabola has symmetry the maximum occurs halfway between 0 and 100 which is 50 sec.

If the maximum occurs at 50 sec. find the height by plugging 50 into the height equation H(50)-1600(50)-16$(50)^2$=40,000 ft.

Trial and Error Method:

Guess and check values should test 50 as well as numbers on either side of 50. Other answers to part B and C must be explained.

Graphic Method:

Using graphing utility

Put $1600t-16t^2$ into y= and graph. Use max function to determine the y-value. Use zero function to determine when the missile hits the ground. Use trace to also find maximum height or time. Set window to view parabola.
[-10, 100] by [-100, 50,000]

Graphical solutions must explain what key sequences/methods are used to obtain the values.

5 ADVANCED UNDERSTANDING – EXCELLENT

Response includes correct numerical answers A) 40,000 ft, B) 50 sec., C) 100 sec.
All work shown and fully explained.
Explanation includes ***why*** steps were performed.
Nothing incorrect (All quantities have appropriate labels).

4 SATISFACTORY UNDERSTANDING

Response includes correct numerical answers A) 40,000 ft., B) 50 sec., C) 100 sec.
All work shown and/or explained.
Some explanation required.
Explanation of why steps were performed may be incomplete, unclear, or might explain what is being done, but an attempt is made.
Work and/or explanation may contain minor blemishes.

3 ALMOST SATISFACTORY UNDERSTANDING

A.) Response includes correct answers A) 40,000 ft., B) 50 sec., C) 100 sec.
All work shown with no explanation.
Work may contain minor blemishes

or

B.) Response includes correct answers.
Adequate work shown or explained (some steps are missing but you can follow what is being done – all appropriate mathematical procedures are used.)

or

C.) Response includes incorrect numerical answer due to one calculation, copying, or rounding error.
Adequate work shown and/or explained.

2 PARTIAL UNDERSTANDING

A.) Response includes correct numerical answers.
Some work and/or explanation
Appropriate mathematical procedures used.

or

B.) Response includes incorrect numerical answer due to more than one calculation or copying errors.
Adequate work shown or explained.

or

C.) Response includes procedural error (ex: dividing equation by t and losing a root).

or

D.) Response shows correct answers to two parts with some work/explanation.

1 MINIMAL UNDERSTANDING

A.) Response includes correct numerical answers.
No supporting work or explanation.

or

B.) Response shows correct answer to one part with work or explanation.

or

C.) Response shows factoring of equation and solution for x=0 and x=100 but don't do anything with them.

0 INCORRECT

A.) Work not meeting minimal requirements for score point of 1 or above. Ex.) Factor $1600t-16t^2=16t(100-t)$ but does noting else, not set = to 0.

or

B.) Correct answers with conflicting work or explanation.
Response contains nothing correct.
Factors $1600t-16t^2$ and finds roots of x=0 and x=100 and uses them incorrectly in the problem. Ex) H=100 ft for maximum height; H=0 ft for maximum height; x=100 ft where missile reaches maximum.

The height, in feet, of a missile is given by $H = 1600t - 16t^2$ and t is the time in seconds.

A) What is the maximum height reached by the missile?
B) What is the time it reaches the maximum height
C) What is the time when the missile hits the ground

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AND

2. write an explanation stating the mathematical reason(s) **why** you chose each of your steps.

$H = 1600t - 16t^2$

$a = -16 \quad b = 1600 \quad c = 0$

① $x = \frac{-b}{2a} = \frac{-1600}{2(-16)} = 50 \text{ sec}$

② $H = 1600(50) - 16(50)^2$

$= 80{,}000 - 40{,}000$

$H = 40{,}000 \text{ Ft}$

③ $x = \frac{-1600 \pm \sqrt{(1600)^2 - 4(-16)(0)}}{2(-16)}$

$x = \frac{-1600 \pm \sqrt{2560000}}{-32}$

$x = \frac{-1600 \pm 1600}{-32}$

$x = 0 \text{ sec}$

$x = 100 \text{ sec}$

① Because it is a parabola, you can use the line of symmetry to find the time when the missile reaches the maximum height. 50 seconds is when it reaches maximum.

② Put 50 into the equation to get the height, the maximum height is 40,000 ft.

③ Put into the quadratic formula to find the time. $x = 0$ sec is before the missile leaves the ground $x = 100$ sec is when it hits the ground.

A) 40,000 Ft B) 50 sec C) 100 sec

5 – Student shows values for a, b, c to use in formulas. Student uses appropriate units. All work explained with why.

The height, in feet, of a missile is given by $H = 1600t - 16t^2$ and t is the time in seconds.

A) What is the maximum height reached by the missile? 40 000ft
B) What is the time it reaches the maximum height 50s
C) What is the time when the missile hits the ground 100 s

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A) $H = 1600t - 16t^2$

$H = 1600(1) - 16(1^2)$ $H = 1600(2) - 16(2)^2$ $H = 1600(3) - 16(3)^2$ $H = 1600(4) - 16(4)^2$

$H = 1600 - 16$ $H = 3200 - 64$ $H = 4800 - 144$ $H = 6400 - 256$

$H = 1584$ ft $H = 3136$ ft $H = 4656$ ft $H = 6144$ ft

$H = 1600(5) - 16(5)^2$ $H = 1600(6) - 16(6)^2$ $H = 1600(7) - 16(7)^2$ $H = 1600(8) - 16(8)^2$

$H = 8000 - 400$ $H = 9600 - 576$ $H = 11200 - 784$ $H = 12800 - 1024$

$H = 7600$ ft $H = 8424$ ft $H = 10416$ ft $H = 11776$ ft

$H = 1600(9) - 16(9)^2$ $H = 1600(10) - 16(10)^2$ $H = 1600(11) - 16(11)^2$ $H = 1600(50) - 16(50)^2$

$H = 14400 - 1296$ $H = 16000 - 1600$ $H = 17600 - 1936$ $H = 80000 - 40000$

$H = 13104$ ft $H = 14400$ ft $H = 15664$ ft $\therefore H = 40000$ ft

$H = 1600(51) - 16(51)^2$

$H = 81600 - 41616$

a decrease $H = 39984$ ft

I substituted values of t into the equation in a attempt to find max. height, time of max height, & when missile hits ground. By substituting 100 in for t ($H = 1600(100) - 16(100)^2$) you get $H = 0$ ft, so that is the time it hits the ground (100s). After a while I got bored of substituting every #, so I started to guess. when I tried 50s, I got 40000 ft = H. Then I tried 51s & got H = 39984 ft. To double check, I tried 49s & got 39984 ft = H. Therefore, I knew the max. height was 40000 ft & the time at which this occurred was 50 s, b/c that's the t value I substituted in

5 – Students do not have to do this many trials, but should show trials on either side to verify the maximum.

The height, in feet, of a missile is given by $H = 1600t - 16t^2$ and **t** is the time in seconds.

A) What is the maximum height reached by the missile?
B) What is the time it reaches the maximum height
C) What is the time when the missile hits the ground

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$1600 - 32t = H'$
$-32t = -1600$
$t = 50$ sec.

I used the derivative of the equation to find the value of t. I then put the value I got for t back into the original equation to get height

$H = 1600(50) - 16(50)^2$
$H =$ 40,000 ft. max height

$-32 = H'$ → I took the derivative of the equation I developed to tell whether the number I got for height was the maximum or minimum. The negative sign tells me it is the maximum

time when reaches max height 50 sec.
time when missile hits ground 1 min 40 sec.

$H = 1600(100) - 16(100)^2$
$H = 0$
at 100 secs the height of the missle is zero which means it is back on the ground

5 – Student identifies why 50 is a maximum.

The height, in feet, of a missile is given by $H = 1600t - 16t^2$ and t is the time in seconds.

A) What is the maximum height reached by the missile?
B) What is the time it reaches the maximum height
C) What is the time when the missile hits the ground

For full credit, you **must** do the following:

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A. $t=5$ $1600t - 16t^2$

1584

$1600(50) - 16(50)^2$

$80{,}000 - 40000 = 40{,}000$

$1600(49) - 16(49)^2$

$78400 - 38416 = 39984$ ✓

$1600(51) - 16(51)^2$

$81600 - 41616 = 39984$ ✓

A = 40,000 ft.

B. 50 sec.

8000

C. Ground: height = 0.

$0 = 1600t - 16t^2$

$0 = 16t(100 - t)$

$0 = 100 - t$

$t = 100$

100 sec when missile hits ground

4 – All work shown but explanation is needed to attain a 5.

The height, in feet, of a missile is given by $H = 1600t - 16t^2$ and t is the time in seconds.

A) What is the maximum height reached by the missile?
B) What is the time it reaches the maximum height
C) What is the time when the missile hits the ground

For full credit, you **must** do the following:

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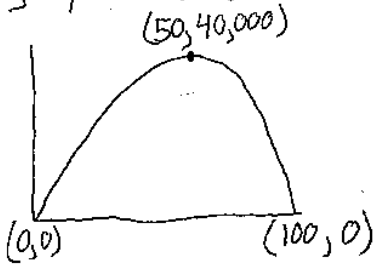
AND

2. write an explanation stating the mathematical reason(s) **why** you chose each of your steps.

1) graph on calc



Max height = 40,000 ft
time Max height = 50 sec
time to hit ground = 100 sec

2) I graphed it on the calculater.

I used the trace button on the calculater and found the maximum height, the time it takes to reach the maximum height, and the time it takes to hit the ground.

4 – Axes of the graph are not labeled with time and height. Trace is explained but how you put it in the calculator and the window should also be explained.

The height, in feet, of a missile is given by $H = 1600t - 16t^2$ and t is the time in seconds.

A) What is the maximum height reached by the missile?
B) What is the time it reaches the maximum height
C) What is the time when the missile hits the ground

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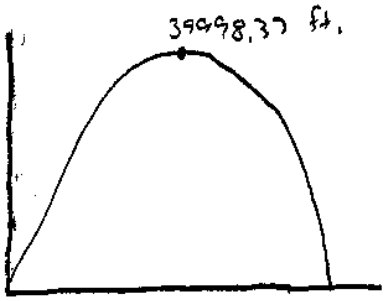

[-10, 100] [-100, 50000] set the window @

then tell the calc. to give you the max value which

A) is 39998.3761.

$H = -16(100t - t) = 0$ solve for "t"

C) $t = 100$ sec

B) Sec to max = 50 sec, divide "t" by 2.

3 – Rounding error in part A (graphical solutions should be using max) and answers should be exact.

The height, in feet, of a missile is given by $H = 1600t - 16t^2$ and t is the time in seconds.

A) What is the maximum height reached by the missile?
B) What is the time it reaches the maximum height
C) What is the time when the missile hits the ground

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$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$ $\frac{-1600 \pm \sqrt{(1600)^2-4(-16)(0)}}{2(-16)}$

$\frac{-1600 \pm 1600}{-32} = 0$ 100 seconds total time

A. 40,000 ft
B. 50 seconds
C. 100 seconds

I found the total time first using the quadratic equation. The total vertical displacement is 0 so I found the time to be 100 seconds. Since the max height is the same up and down, acceleration from gravity is the same so the time would be the same. So I divided the total time by two, 50 seconds. I plugged that into the equation and found the max height to be 40,000 ft.

3 – Adequate work shown or explained. Should show input into equations to find part A answer.

The height, in feet, of a missile is given by $H = 1600t - 16t^2$ and t is the time in seconds.

A) What is the maximum height reached by the missile?
B) What is the time it reaches the maximum height
C) What is the time when the missile hits the ground

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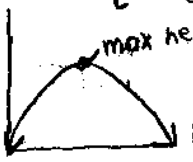
$H = 1600t - 16t^2$



Find the maximum and minimum:
(1) Do derivative
(2) Find critical points
(3) Determine maximum

$y = 4p(x-h)$
$x = 4p(y-k)$

Complete the square:

$H = 16t(100 - t)$

$16t = 0 \quad 100 = t$
$t \neq 0 \quad t = 100$ seconds

$H' = 1600 - 32t \rightarrow$ derivative

$1600 = 32t$
$t = 50$ seconds

2 – No answer to part A. Could use more explanation or labeling of the parts.

The height, in feet, of a missile is given by $H = 1600t - 16t^2$ and t is the time in seconds.

A) What is the maximum height reached by the missile?
B) What is the time it reaches the maximum height
C) What is the time when the missile hits the ground

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$Y = 1600x - 16x^2$ - GRAPHED

OPENED TABLE

ALL ANSWERS FOUND IN TABLE.

A.) 40,000ft

B.) 50 SECONDS

C.) 100 SECONDS

2 – Not enough explanation of the method.

The height, in feet, of a missile is given by $H = 1600t - 16t^2$ and t is the time in seconds.

A) What is the maximum height reached by the missile?
B) What is the time it reaches the maximum height
C) What is the time when the missile hits the ground

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$H = 1600t - 16t^2$ $\frac{H}{T} = 1600 - 16t$

To achieve maximum height, I want 1600t to be the greatest possible number while $16t^2$ is the least possible number. First, I find a pattern.

t	H
1	1584
2	3036
30	33600
70	33,600

missile rising
missile falling

H will continue to be higher until the time is so great that the missile is falling.

max ht occur when $T = \frac{30+70}{2} = 50$.

A $H = 50(1600) - 16(50)^2 = 40,000$ ft

B. max ht occurs at $t = 50$

The object crashes into the ground when $1600t = 16t^2$ $t = 100$

Simplify $1600 = 16t$ C, when $t = 100$, missile hits ground

2 – Shows using midpoint concept to find the *x* coordinate of max. Conceptual error divided by the *t* and lost the root of 0. Units missing on B and C.

The height, in feet, of a missile is given by $H = 1600t - 16t^2$ and t is the time in seconds.

A) What is the maximum height reached by the missile?
B) What is the time it reaches the maximum height
C) What is the time when the missile hits the ground

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AND

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A: Max H = 40,000 $1600t - 16t^2 = H$

B: Time = 50 Seconds Solve T

C. Time = 100 Seconds

1 – No work.

The height, in feet, of a missile is given by $H = 1600t - 16t^2$ and t is the time in seconds.

A) What is the maximum height reached by the missile?
B) What is the time it reaches the maximum height
C) What is the time when the missile hits the ground

For full credit, you **must** do the following:

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AND

2. write an explanation stating the mathematical reason(s) **why** you chose each of your steps.

$H = 1600t - 16t^2$

$-16t^2 + 1600t + 0 = 0$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1600 \pm \sqrt{1600^2 - 4(-16)(0)}}{2(-16)} = \frac{-1600 \pm \sqrt{2560000}}{-32}$

$= \frac{(+)}{} \frac{3200}{32} = 100$ OR $= \frac{(-)}{} \frac{0}{32} = 0$

$t = 100$ or 0

A) $H = 1600t - 16t^2 = 1600(100) - 16(100^2) = 0$

$H = 0$

To find "t", I used the quadratic formula ($x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$) and found that t could either be 100 sec or 0 sec. Then by plugging "t" back into the equation to find "H", I found that both of the values of "t" ended up giving me H = 0. This perplexes me. It seems that A) the max height = 0 B) the max time is 100s and C) the missile never left the ground. ???

1 – Finds the zeros but doesn't do anything with it.

The height, in feet, of a missile is given by $H = 1600t - 16t^2$ and **t** is the time in seconds.

A) What is the maximum height reached by the missile?
B) What is the time it reaches the maximum height
C) What is the time when the missile hits the ground

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$H = 1,600t - 16t^2$ It will reach
$+ 16t^2 \quad + 16t^2$ the height in
100 seconds.

$\frac{16t^2}{16t^2} = \frac{1,600t}{16t^2}$ $t = 100$.

It will reach the max. height in 16 seconds. That is also the time it will hit the ground.
I added 16 to both sides so I could get it alone then divided to get my answer
$t = 100$.

0 – Should not have divided by t^2. Incorrectly interprets t=100 two times. Equates time for maximum height with time to hit the ground.