Unit 1: Relations, Functions, and Transformations

Course Time Prior to Keystone/PSSA:
- Periods – 12
- Blocks – 6

Overview: Student will be able to
- Graph relations and functions and determine the domain and range of each.
- Model real-world data and make future predictions using scatter plots.
- Transform absolute value functions.

Unit Essential Questions:
- How do you use transformations to help graph absolute value functions?
- How can you model data with a linear function?

Keystone Content Module/Assessment Anchor:
- **A2.1.3.2** Describe and/or determine change.
- **A2.2.1.1** Analyze and/or use patterns or relations
- **A2.2.2.2** Describe and/or determine families of functions
- **A2.2.2.2** Analyze and/or interpret data on a scatter plot and/or use a scatter plot to make predictions

Keystone Eligible Content:
- **A2.1.3.2.1** Determine how a change in one variable relates to a change in a second variable (e.g., y = 4/x; if x doubles, what happens to y?).
- **A2.1.3.2.2** Use algebraic processes to solve a formula for a given variable (e.g., solve d = rt for r).
- **A2.2.1.1.1** Analyze a set of data for the existence of a pattern and represent the pattern with a rule algebraically and/or graphically
- **A2.2.1.1.3** Determine the domain, range, or inverse of a relation.
- **A2.2.2.2.1** Identify or describe the effect of changing parameters within a family of functions (e.g., $y = x^2$ and $y = x^2 + 3$, or $y = x^2$ and $y = 3x^2$).
- **A2.2.3.1.1** Draw, identify find, interpret, and/or write an equation for a regression model (lines and curves of best fit) for a scatter plot.
- **A2.2.3.1.2** Make predictions using the equations or graphs of regression models (lines and curves of best fit) of scatter plots.

Pennsylvania Common Core Standard(s):
- **CC.2.1.HS.F.3** Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs and data displays.
- **CC.2.1.HS.F.5** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
- **CC.2.2.HS.C.1** Use the concept and notation of functions to interpret and apply them in terms of their context.
- **CC.2.2.HS.C.2** Graph and analyze functions and use their properties to make connections between the different

- **CC.2.2.HS.C.3** Write functions or sequences that model relationships between two quantities.
- **CC.2.2.HS.C.5** Construct and compare linear, quadratic and exponential models to solve problems.
- **CC.2.2.HS.C.6** Interpret functions in terms of the situation they model.
- **CC.2.4.HS.B.2** Summarize, represent, and interpret data on two categorical and quantitative variables.
- **CC.2.4.HS.B.3** Analyze linear models to make interpretations based on the data.

National Common Core Standard(s):
Build new functions from existing functions.
- **CC.9-12.F.BF.3** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

Understand the concept of a function and use function notation.
- **CC.9-12.F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
- **CC.9-12.F.IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Interpret functions that arise in applications in terms of the context.
- **CC.9-12.F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
- **CC.9-12.F.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations.
- **CC.9-12.F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
  b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

ISTE Standards:
1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
    b. Create original works as a means of personal or group expression
    c. Use models and simulations to explore complex systems and issues
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
    a. Interact, collaborate, and publish with peers, experts, or others employing a variety of digital environments and media
    b. Communicate information and ideas effectively to multiple audiences using a variety of media and formats
    d. Contribute to project teams to produce original works or solve problems
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.

a. Plan strategies to guide inquiry
b. Locate, organize, analyze, evaluate, synthesize, and ethically use information from a variety of sources and media
c. Evaluate and select information sources and digital tools based on the appropriateness to specific tasks
d. Process data and report results
4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
a. Identify and define authentic problems and significant questions for investigation
b. Plan and manage activities to develop a solution or complete a project
c. Collect and analyze data to identify solutions and/or make informed decisions
d. Use multiple processes and diverse perspectives to explore alternative solutions
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.
a. Advocate and practice safe, legal, and responsible use of information and technology
b. Exhibit a positive attitude toward using technology that supports collaboration, learning, and productivity
c. Demonstrate personal responsibility for lifelong learning
d. Exhibit leadership for digital citizenship
6. Technology Operations and Concepts – Students demonstrate a sound understanding of technology concepts, systems, and operations.
a. Understand and use technology systems
b. Select and use applications effectively and productively

Career Education and Work Standards:
- 13.1.11.A   Relate careers to individual interests, abilities, and aptitudes.
- 13.1.11.E   Justify the selection of a career.
- 13.1.11.H   Review personal high school plan against current personal career goals and select postsecondary opportunities based upon personal career interests.

Connecting to Common Core and Other Standards:
PA Standards found at www.pdesas.org/standards/standardsdownloads
National Common Core found at www.corestandards.org
ISTE found at www.iste.org/standards/nets-for-students.aspx
Career Education and Work found at www.pacareerstandards.com/
*See Appendix for complete documents.

ELL Differentiation: Math & LA specifics found at www.pde.sas.org/module/sas/curriculumframework/elloverlay.aspx
Generic found at http://www.easad.net/esl
Todos, found at www.todos-math.org

Enrichment:
- Piecewise functions
- Collect data to create a self-constructed scatter plot
- Internet/Research Activities

- o www.usatestprep.com
- o www.studyisland.com
- o www.thelearningodyssey.com
- o www.hippocampus.org
- Group/Research projects

Remediation:
- Common terms from Algebra I: x-axis, y-axis, x-intercept, y-intercept, Coordinate Plane, Cartesian Plane, etc.
- Graph and identify relations and functions.
- Apply and identify properties of real numbers to evaluate and simplify algebraic expressions.
- Use variables to represent unknown quantities in real-world situations.
- Solve equations, inequalities and absolute values equations with one variable.
- Solve problems with lines using slope-intercept form, point-slope form, and standard form.

IEP/GIEP: Refer to individual student's education plan under specially designed instruction.

Keystone Anchors & Eligible Content	Unit Concepts What students need to know	Unit Competencies What students need to be able to do (skills): (Students will:)	Content Vocabulary	Materials, Resources, & Instructional Activities	Assessments
A2.1.3.2 A2.1.3.2.2 A2.2.1.1 A2.2.1.1.1 A2.2.1.1.3 A2.2.2.2 A2.2.2.2.1 A2.2.3.1 A2.2.3.1.1 A2.2.3.1.2	• Determine how a change in one variable relates to a change in a second variable • Use algebraic processes to solve a formula for a given variable • Analyze and/or use patterns or relations • Describe and/or determine families of functions • Analyze and/or interpret data on a scatter plot and/or use a scatter plot to make predictions • Differentiate between relations and functions • Define direct variation • Graph and model linear equations • Writing equations of lines given slope and a point • Create scatter plots and use them to make future predictions • Transformation of functions • Graphing absolute	• Analyze a set of data for the existence of a pattern and represent the pattern with a rule algebraically and/or graphically • Classify and evaluate functions • Use function notation while solving problems. • Determine the domain, range, or inverse of a relation. • Identify or describe the effect of changing parameters within a family of functions • Draw, identify find, interpret, and/or write an equation for a regression model (lines and curves of best fit) for a scatter plot. • Make predictions using the equations or graphs of regression models (lines and curves of best fit) of scatter plots. • Write equations and	• Absolute Value Function • Correlation • Direct Variation • Function • Families of Functions • Parent Function • Function Notation • Linear Function • Linear Equation • Linear Inequality • Domain • Range • Interval Notation • Extraneous Solution • Relation • Inverse Relation • Relation • Mapping • Slope • Pattern & Pattern Rule • Scatter Plot • Line of Best Fit (or Best Fit Line) • Curve of Best Fit (or Best Fit Curve) • Rate of Change • Slope • Transformations • Regression Model • Regression Curve • Coordinate Plane	• www.usatestprep.com • www.hippocampus.org • www.classzone.com • www.phsuccessnet.com • www.StudyIsland.com • https://www.thelearningodyssey.com/ • Algebra 2, Concepts and Skills, (McDougal Littell, 2004) • Algebra 2, (McDougal Littell, 2008) • Algebra 2, Prentice Hall, 2007 • Common Core Algebra 2,(Pearson 2012) • OnCore Mathematics, Algebra 2, (Houghton Mifflin Harcourt, 2010) • McDougal Littell & Prentice Hall teaching resources • Calculators • CPS Clickers • YouTube • ESASD Valuable Video Library • New & updated resources available	• Tests/quizzes • CDTs • Compass Learning • Study Island • USA Test Prep • Observation • Homework • Class Work • Projects • PSSA Problems • Journals • Bell Ringers

	value functions • Graphing two-variable inequalities	inequalities to represent word problems relating to real-world situations	• x-axis • y-axis • x-intercept(s) • y-intercept(s) • Variable	on departmental page	

Unit 2: Linear Systems

Course Time Prior to Keystone/PSSA:
- Periods – 16
- Blocks – 8

Overview: Student will be able to
- Demonstrate competency in the following: graphing linear equations, writing the equation of a line, computing the slope of a line, solving an equation for a variable, identifying functions, graphing 2-variable inequalities, and graphing absolute value functions and transformations.
- Solve a system of linear equations by graphing the equations to find the point(s) of intersection.
- Determine whether a system has one solution, no solutions, or infinitely many solutions.
- Use substitution and elimination methods to write equivalent equations until they get an equation with only one variable.
- Solve systems of linear inequalities.
- Solve problems using linear programming.

Unit Essential Questions:
- How does representing functions graphically help you solve a system of equations?
- How does writing equivalent equations help you solve a system of equations?
- How does linear inequalities help solve linear programming problems?

Keystone Content Module/Assessment Anchor:
- **A2.2.1.1** Analyze and/or use patterns or relations.

Keystone Eligible Content:
- **A2.2.1.1.1** Analyze a set of data for the existence of a pattern and represent the pattern with a rule algebraically and/or graphically.

Pennsylvania Common Core Standard(s):
- **CC.2.1.HS.F.7** Apply concepts of complex numbers in polynomial identities and quadratic equations to solve problems.
- **CC.2.2.HS.C.1** Use the concept and notation of functions to interpret and apply them in terms of their context.
- **CC.2.2.HS.C.2** Graph and analyze functions and use their properties to make connections between the different representations.
- **CC.2.2.HS.C.3** Write functions or sequences that model relationships between two quantities.
- **CC.2.2.HS.C.5** Construct and compare linear, quadratic and exponential models to solve problems.
- **CC.2.2.HS.C.6** Interpret functions in terms of the situation they model.
- **CC.2.4.HS.B.2** Summarize, represent, and interpret data
- **CC.2.2.8.B.3**   Analyze and solve linear equations and pairs of simultaneous linear equations.
- **CC.2.1.HS.F.4** Use units as a way to understand problems and to guide the solution of multi-step problems.
- **CC.2.1.HS.F.5** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
- **CC.2.2.HS.D.7** Create and graph equations or inequalities to describe numbers or relationships.

- **CC.2.2.HS.D.8** Apply inverse operations to solve equations or formulas for a given variable.
- **CC.2.2.HS.D.9** Use reasoning to solve equations and justify the solution method.

National Common Core Standard(s):

Create equations that describe numbers or relationships.

- **CC.9-12.A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **CC.9-12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

Solve systems of equations.

- **CC.9-12.A.REI.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Represent and solve equations and inequalities graphically.

- **CC.9-12.A.REI.12** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Perform operations on matrices and use matrices in applications.

- **CC.9-12.N.VM.6** Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
- **CC.9-12.N.VM.7** Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
- **CC.9-12.N.VM.8** Add, subtract, and multiply matrices of appropriate dimensions.
- **CC.9-12.N.VM.9** Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
- **CC.9-12.N.VM.10** Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
- **CC.9-12.N.VM.11** Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
- **CC.9-12.N.VM.12** Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

ISTE Standards:

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   b. Create original works as a means of personal or group expression
   c. Use models and simulations to explore complex systems and issues
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
   a. Interact, collaborate, and publish with peers, experts, or others employing a variety of digital environments and media
   b. Communicate information and ideas effectively to multiple audiences using a variety of media and formats
   d. Contribute to project teams to produce original works or solve problems
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.

a. Plan strategies to guide inquiry
b. Locate, organize, analyze, evaluate, synthesize, and ethically use information from a variety of sources and media
c. Evaluate and select information sources and digital tools based on the appropriateness to specific tasks
d. Process data and report results
4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
   a. Identify and define authentic problems and significant questions for investigation
   b. Plan and manage activities to develop a solution or complete a project
   c. Collect and analyze data to identify solutions and/or make informed decisions
   d. Use multiple processes and diverse perspectives to explore alternative solutions
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.
   a. Advocate and practice safe, legal, and responsible use of information and technology
   b. Exhibit a positive attitude toward using technology that supports collaboration, learning, and productivity
   c. Demonstrate personal responsibility for lifelong learning
   d. Exhibit leadership for digital citizenship
6. Technology Operations and Concepts – Students demonstrate a sound understanding of technology concepts, systems, and operations.
   a. Understand and use technology systems
   b. Select and use applications effectively and productively

Career Education and Work Standards:
- 13.1.11.A   Relate careers to individual interests, abilities, and aptitudes.
- 13.1.11.E   Justify the selection of a career.
- 13.1.11.H   Review personal high school plan against current personal career goals and select postsecondary opportunities based upon personal career interests.

Connecting to Common Core and Other Standards:
PA Standards found at www.pdesas.org/standards/standardsdownloads
National Common Core found at www.corestandards.org
ISTE found at www.iste.org/standards/nets-for-students.aspx
Career Education and Work found at www.pacareerstandards.com/
*See Appendix for complete documents.

ELL Differentiation: Math & LA specifics found at www.pde.sas.org/module/sas/curriculumframework/elloverlay.aspx
Generic found at http://www.easad.net/esl
Todos, found at www.todos-math.org

Enrichment:
- Graph points in a 3D plane
- Perform basic matrix operations
- Solve systems of equations using matrices
- Solve real-world problems using matrices
- Solve a three-variable system using elimination
- Solve a three-variable system using substitution
- Internet/Research Activities
    - www.usatestprep.com
    - www.studyisland.com
    - www.thelearningodyssey.com
    - www.hippocampus.org
- Group/Research projects

Remediation:
- Graph linear equations and inequalities
- Solve an equation for a variable
- Evaluate algebraic expressions
- Write linear equations in slope-intercept form

IEP/GIEP: Refer to individual student's education plan under specially designed instruction.

Keystone Anchors & Eligible Content	Unit Concepts What students need to know	Unit Competencies What students need to be able to do (skills): (Students will:)	Content Vocabulary	Materials, Resources, & Instructional Activities	Assessments
A2.2.1.1 A2.2.1.1.1	• Solve a system of equations by graphing, substitution, and elimination • Identify and use the three classifications of systems of equations: independent, dependent, and inconsistent. • Solve a system of inequalities by graphing • Model real-world data and solve using linear programming • Solve systems in three variables using elimination and substitution • Analyze and/or use patterns or relations	• Classify a system of equations as dependent, independent, consistent, or inconsistent • Identify coincident lines • Solve a system of equations by graphing • Solve a system of equations using substitution • Solve a system of equations using elimination • Solve an equivalent system using a graph • Use linear programming to solve problems • Find and/or calculate the minimum/maximum values of the objective function • Solve a system of inequalities by graphing • Identify the restrictions on the variables in a problem-solving situation • Identify the vertices of the graph • Analyze a set of data for the existence of a pattern and represent the pattern with a rule algebraically and/or graphically • Make real-world connections while problem-solving	• Dependent System • Equivalent Systems • Independent Systems • Linear Systems • Coincident Lines • System of Linear Equations • System of Linear Inequalities • Substitution Method • Elimination Method • Graph or Graphing Equations • Consistent • Inconsistent • Ordered Pair • Ordered Triple • x-Intercept • y-Intercept • z-Intercept • Solution • Solution Set • Empty or Null Set • Infinitely Many Solutions • Relations • Patterns • Linear Combination • Linear Programming • Objective Function • Constraints • Feasible Region	• www.usatestprep.com • www.hippocampus.org • www.classzone.com • www.phsuccessnet.com • www.StudyIsland.com • https://www.thelearning odyssey.com/ • Algebra 2, Concepts and Skills, (McDougal Littell, 2004) • Algebra 2, (McDougal Littell, 2008) • Algebra 2, Prentice Hall, 2007 • Common Core Algebra 2,(Pearson 2012) • OnCore Mathematics, Algebra 2, (Houghton Mifflin Harcourt, 2010) • McDougal Littell & Prentice Hall teaching resources • Calculators • CPS Clickers • YouTube • ESASD Valuable Video Library • New & updated resources available on departmental page	• Tests/quizzes • CDTs • Compass Learning • Study Island • USA Test Prep • Observation • Homework • Class Work • Projects • PSSA Problems • Journals • Bell Ringers

		• Write and use linear models to represent real-world situations	• Inverse Operations • Parallel Lines • Skew Lines • Perpendicular Lines	

Unit 3: Quadratic Functions & Equations and Complex Numbers

Course Time Prior to Keystone/PSSA:
- Period – 24
- Block – 12

Overview: Student will be able to:
- Identify and graph quadratic functions.
- Graph quadratic functions and their transformations.
- Find the zeros of a quadratic function by graphing, factoring, completing the square, and using the Quadratic Formula.
- Identify, graph, and perform operations with complex numbers, as well as find complex number solutions of quadratic equations.

Unit Essential Questions:
- What are the advantages of a quadratic function in vertex and standard forms?
- How is any quadratic function related to the parent quadratic function $y = x^2$?
- How are the real solutions of a quadratic equation related to the graph of the related quadratic function?

Keystone Content Module/Assessment Anchor:
- **A2.1.1.1** Represent and/or use imaginary numbers in equivalent forms (e.g., square roots and exponents).
- **A2.1.1.2** Apply the order of operations in computation and in problem-solving situations.
- **A2.1.2.1** Use exponents, roots, and/or absolute values to represent equivalent forms or to solve problems.
- **A2.1.3.1** Write and/or solve non-linear equations using various methods.
- **A2.2.1.1** Analyze and/or use patterns or relations.
- **A2.2.2.1** Create, interpret, and/or use polynomial, exponential, and/or logarithmic functions and their equations, graphs, or tables.
- **A2.2.2.2** Describe and/or determine families of functions.

Keystone Eligible Content:
- **A2.1.1.1.1** Simplify/write square roots in terms of i (e.g., $\sqrt{-24} = 2i\sqrt{6}$).
- **A2.1.1.1.2** Simplify/evaluate expressions involving powers of i (e.g., $i^6 + i^3 = -1 - i$).
- **A2.1.1.2.1** Add and subtract complex numbers (e.g., $(7 - 3i) - (2 + i) = 5 - 4i$).
- **A2.1.1.2.2** Multiply and divide complex numbers (e.g., $(7 - 3i)(2 + i) = 17 + i$).
- **A2.1.2.1.1** Use exponential expressions to represent rational numbers.
- **A2.1.2.1.2** Simplify/evaluate expressions involving positive and negative exponents and/or roots (may contain all types of real numbers – exponents should not exceed power of 10).
- **A2.1.2.1.3** Simplify/evaluate expressions involving multiplying with exponents (e.g., $x^6 \cdot x^7 = x^{13}$), powers of powers (e.g., $(x^6)^7 = x^{42}$) and powers of products (e.g., $(2x^2)^3 = 8x^6$). <u>Note</u>: Limit to rational exponents.
- **A2.1.3.1.1** Write and/or solve quadratic equations (including factoring and using the Quadratic Formula).
- **A2.1.3.1.2** Solve equations involving rational and/or radical expressions (e.g., $10/(x + 3) + 12/(x - 2) = 1$ or $\sqrt{x^2 + 21x} = 14$).

- **A2.1.3.1.3** Write and/or solve a simple exponential equation.
- **A2.2.1.1.4** Identify and/or determine the characteristics of an exponential, quadratic, or polynomial function (e.g., intervals of increase/decrease, intercepts, zeros, and asymptotes).
- **A2.2.2.1.1** Create, interpret, and/or use the equation, graph, or table of a polynomial function (including quadratics).
- **A2.2.2.1.2** Create, interpret, and/or use the equation, graph, or table of an exponential function.
- **A2.2.2.1.3** Determine, use, and/or interpret minimum and maximum values over a specified interval of a graph of
- a polynomial or exponential function.
- **A2.2.2.1.4** Translate a polynomial and/or exponential function from one representation of a function to another (graph, table, and equation).
- **A2.2.2.2.1** Identify or describe the effect of changing parameters within a family of functions. (e.g., $y = x^2$ and $y = x^2 + 3$, or $y = x^2$ and $y = 3x^2$).

Pennsylvania Common Core Standard(s):
- **CC.2.1.HS.F.1** Apply and extend the properties of exponents to solve problems with rational exponents.
- **CC.2.1.HS.F.3** Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs, and data displays.
- **CC.2.1.HS.F.4** Use units as a way to understand problems and to guide the solution of multi-step problems.
- **CC.2.1.HS.F.6** Extend the knowledge of arithmetic operations and apply to complex numbers.
- **CC.2.1.HS.F.7** Apply concepts of complex numbers in polynomial identities and quadratic equations to solve problems.
- **CC.2.2.HS.C.2** Graph and analyze functions and use their properties to make connections between the different
- **CC.2.2.HS.C.3** Write functions or sequences that model relationships between two quantities.
- **CC.2.2.HS.C.4** Interpret the effects transformations have on functions and find the inverses of functions.
- **CC.2.2.HS.C.5** Construct and compare linear, quadratic and exponential models to solve problems.
- **CC.2.2.HS.C.6** Interpret functions in terms of the situation they model.
- **CC.2.2.HS.D.2** Write expressions in equivalent forms to solve problems.
- **CC.2.2.HS.D.5** Use polynomials identities to solve problems.
- **CC.2.2.HS.D.6** Extend the knowledge of rational functions to rewrite in equivalent forms.
- **CC.2.2.HS.D.7** Create and graph equations or inequalities to describe numbers or relationships.
- **CC.2.2.HS.D.8** Apply inverse operations to solve problems or formulas for a given variable.
- **CC.2.2.HS.D.9** Use reasoning to solve equations and justify the solution method.
- **CC.2.2.HS.D.10** Represent, solve and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.
- **CC.2.4.HS.B.2** Summarize, represent, and interpret data on two categorical and quantitative variables.

National Common Core Standard(s):
Understand the relationship between zeros and factors of polynomials.
- **CC.9-12.A.APR.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Create equations that describe numbers or relationships.
- **CC.9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

Interpret the structure of expressions.
- **CC.9-12.A.SSE.2** Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Build new functions from existing functions.
- **CC.9-12.F.BF.3** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

Interpret functions that arise in applications in terms of the context.
- **CC.9-12.F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*
- **CC.9-12.F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*
- **CC.9-12.F.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations.
- **CC.9-12.F.IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- **CC.9-12.F.IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Solve equations and inequalities in one variable.
- **CC.9-12.A.REI.4** Solve quadratic equations in one variable.
    - a. Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
    - b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Perform arithmetic operations with complex numbers.
- **CC.9-12.N.CN.1** Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
- **CC.9-12.N.CN.2** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Use complex numbers in polynomial identities and equations.
- **CC.9-12.N.CN.7** Solve quadratic equations with real coefficients that have complex solutions.
- **CC.9-12.N.CN.8** Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* (x + 2*i*)(x – 2*i*).

ISTE Standards:
1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
    b. Create original works as a means of personal or group expression
    c. Use models and simulations to explore complex systems and issues
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
    a. Interact, collaborate, and publish with peers, experts, or others employing a variety of digital environments and media
    b. Communicate information and ideas effectively to multiple audiences using a variety of media and formats
    d. Contribute to project teams to produce original works or solve problems
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.
    a. Plan strategies to guide inquiry
    b. Locate, organize, analyze, evaluate, synthesize, and ethically use information from a variety of sources and media
    c. Evaluate and select information sources and digital tools based on the appropriateness to specific tasks
    d. Process data and report results
4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
    a. Identify and define authentic problems and significant questions for investigation
    b. Plan and manage activities to develop a solution or complete a project
    c. Collect and analyze data to identify solutions and/or make informed decisions
    d. Use multiple processes and diverse perspectives to explore alternative solutions
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.
    a. Advocate and practice safe, legal, and responsible use of information and technology
    b. Exhibit a positive attitude toward using technology that supports collaboration, learning, and productivity
    c. Demonstrate personal responsibility for lifelong learning
    d. Exhibit leadership for digital citizenship
6. Technology Operations and Concepts – Students demonstrate a sound understanding of technology concepts, systems, and operations.
    a. Understand and use technology systems
    b. Select and use applications effectively and productively

Career Education and Work Standards:
- 13.1.11.A   Relate careers to individual interests, abilities, and aptitudes.
- 13.1.11.E   Justify the selection of a career.

- 13.1.11.H   Review personal high school plan against current personal career goals and select postsecondary opportunities based upon personal career interests.

Connecting to Common Core and Other Standards:
PA Standards found at www.pdesas.org/standards/standardsdownloads
National Common Core found at www.corestandards.org
ISTE found at www.iste.org/standards/nets-for-students.aspx
Career Education and Work found at www.pacareerstandards.com/
*See Appendix for complete documents.

ELL Differentiation: Math & LA specifics found at www.pde.sas.org/module/sas/curriculumframework/elloverlay.aspx
Generic found at http://www.easad.net/esl
Todos, found at www.todos-math.org

Enrichment:
- Identify quadratic data/Interpret the average rate of change of a quadratic function
- Solve quadratic inequalities
- Incorporate fractions into equations
- Internet/Research Activities
  - www.usatestprep.com
  - www.studyisland.com
  - www.thelearningodyssey.com
  - www.hippocampus.org
- Group/Research projects

Remediation:
- Solve linear equations
- Inverse operations
- Solve absolute value inequalities
- Write and graph equations in slope-intercept form
- Identify translations
- Solve systems of equations
- Laws of Exponents
- Order of operations

- Calculate the sum and difference of fractions and rational expressions with like and unlike denominators
- Cross Multiplication and Cross Products
- Finding Least Common Denominators (LCD)

IEP/GIEP: Refer to individual student's education plan under specially designed instruction.

Keystone Anchors & Eligible Content	Unit Concepts What students need to know	Unit Competencies What students need to be able to do (skills): (Students will:)	Content Vocabulary	Materials, Resources, & Instructional Activities	Assessments

A2.1.1.1 A2.1.1.1.1 A2.1.1.1.2
 	
 A2.1.1.2 A2.1.1.2.1 A2.1.1.2.2
 	
 A2.1.2.1 A2.1.2.1.1 A2.1.2.1.2 A2.1.2.1.3
 	
 A2.1.3.1 A2.1.3.1.1 A2.1.3.1.2 A2.1.3.1.3
	• Represent and/or use imaginary numbers in equivalent forms.
	 • Apply the order of operations in computation and in problem-solving situations.
	 • Use exponents, roots, and/or absolute values to represent equivalent forms or to solve problems.
	 • Write and/or solve non-linear equations using various methods.
	 .
		• Identify complex numbers
		 • Simplify/write square roots in terms of i
		 • Simplify/evaluate expressions involving powers of i
		 • Add and subtract complex numbers
		 • Multiply and divide complex numbers
		 • Use exponential expressions to represent rational numbers
		 • Simplify/evaluate expressions involving positive and negative exponents and/or roots (may contain all types of real numbers)
		 • Simplify/evaluate expressions that involve multiplying with exponents, powers of powers, and powers of products.
		 • Recognize quadratic functions and parabolas
		 • Write quadratic equations in
		    o Standard form
		    o Vertex form
		    o Intercept form
		 • Write and/or solve quadratic equations by:
		    o Factoring
		    o Graphing
		    o Finding Square Roots
		    o Completing the Square
		 • Write and/or solve equations using
			• Axis of Symmetry
			 • Vertex (Vertices)
			 • Coefficient
			 • Exponent
			 • Real Numbers
			 • Complex Number
			 • Discriminant
			 • Greatest Common Factor (GCF)
			 • Imaginary Number
			 • Parabola
			 • Quadratic Formula
			 • Quadratic Function
			 • Quadratic Equation
			 • Standard Form
			 • Vertex Form
			 • Intercept Form
			 • Zero Product Property
			 • Zeros of a Function
			 • Imaginary Number
			 • Imaginary Unit i
			 • Exponents
			 • Powers
			 • Roots
			 • Absolute Value
			 • Patterns
			 • Relations
			 • Polynomial Function
			 • Exponential
				• www.usatestprep.com
				 • www.hippocampus.org
				 • www.classzone.com
				 • www.phsuccessnet.com
				 • www.StudyIsland.com
				 • https://www.thelearningodyssey.com/
				 • Algebra 2, Concepts and Skills, (McDougal Littell, 2004)
				 • Algebra 2, (McDougal Littell, 2008)
				 • Algebra 2, Prentice Hall, 2007
				 • Common Core Algebra 2,(Pearson 2012)
				 • OnCore Mathematics, Algebra 2, (Houghton Mifflin Harcourt, 2010)
				 • McDougal Littell & Prentice Hall teaching resources
				 • Calculators
				 • CPS Clickers
				 • YouTube
				 • ESASD Valuable Video Library
				 • New & updated resources available
					• Tests/quizzes
					 • CDTs
					 • Compass Learning
					 • Study Island
					 • USA Test Prep
					 • Observation
					 • Homework
					 • Class Work
					 • Projects
					 • PSSA Problems
					 • Journals
					 • Bell Ringers

			Expression		
		the Quadratic Formula • Solve quadratic equations involving rational and/or radical expressions • Write and/or solve a simple exponential equation	• Exponential Function • Exponential Equation • Families of Functions		
Keystone Anchors & Eligible Content	**Unit Concepts** What students need to know	**Unit Competencies** What students need to be able to do (skills): (Students will:)	**Content Vocabulary** (*continued from above*)	**Materials, Resources, & Instructional Activities**	**Assessments**

A2.2.1.1
A2.2.1.1.4
 	
A2.2.2.1
A2.2.2.1.1
A2.2.2.1.2
A2.2.2.1.3
A2.2.2.1.4
 	
A2.2.2.2
A2.2.2.2.1
	• Analyze and/or use patterns or relations.
	• Create, interpret, and/or use polynomial, exponential, and/or logarithmic functions and their equations, graphs, or tables.
	• Describe and/or determine families of functions.
		• Identify and/or determine the characteristics of an exponential, quadratic, or polynomial functions
		  o Vertex
		  o Axis of symmetry
		  o Intervals of increase
		  o Intervals of decrease
		  o Intercepts
		  o Zeros
		  o Asymptotes
		• Create, interpret, and/or use the equation, graph, or table of a polynomial function (including quadratics) when problem-solving
		• Determine, use, and/or interpret minimum and maximum values over a specified interval of a graph of a polynomial or exponential function
		• Translate a polynomial or exponential function from one representation of the function to another
		  o Graph
		  o Table
		  o Equation
		• Identify or describe the effect of changing parameters within a family of functions
		• Make real-world connections while
			• Interval
			• Intercept(s)
			• Maximum Value
			• Minimum Value
			• Asymptote(s)
			• Zero of a Function
			• Standard Form of a Quadratic Function
			• Parabola
			• Completing the Square
			• Best-Fitting Quadratic Model
			• Monomial
			• Binomial
			• Trinomial
			• Least Common Denominator (LCD)
			• Polynomial
			• Square Root
			• Root of an Equation
			• Radical
			• Radicand
			• Rationalizing the Denominator
			• Conjugates
			• Complex Conjugates
				• www.usatestprep.com
				• www.hippocampus.org
				• www.classzone.com
				• www.phsuccessnet.com
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				• CPS Clickers
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				• ESASD Valuable Video Library
				• New & updated resources available
					• Tests/quizzes
					• CDTs
					• Compass Learning
					• Study Island
					• USA Test Prep
					• Observation
					• Homework
					• Class Work
					• Projects
					• PSSA Problems
					• Journals
					• Bell Ringers

		problem-solving	• Complex Plane		

Unit 4: Polynomials and Polynomial Functions

Course Time Prior to Keystone/PSSA:
- Period – 20
- Block – 10

Overview: Student will be able to:
- Classify and graph polynomial functions.
- Describe the end behavior of a polynomial function and identify relative the relative maximum or minimum of the function.
- Solve polynomial equations by factoring and graphing, as well as divide polynomials using long division and synthetic division.

Unit Essential Questions:
- What does the degree of a polynomial tell you about its related polynomial function?
- For a polynomial function, how are factors, zeros, and x-intercepts related?
- For a polynomial equation, how are factors and roots related?

Keystone Content Module/Assessment Anchor:
- **A2.1.2.1** Use exponents, roots, and/or absolute values to represent equivalent forms or to solve problems.
- **A2.1.2.2** Simplify expressions involving polynomials.
- **A2.1.3.1** Write and/or solve non-linear equations using various methods.
- **A2.2.1.1** Analyze and/or use patterns or relations.
- **A2.2.2.1** Create, interpret, and/or use polynomial, exponential, and/or logarithmic functions and their equations, graphs, or tables.
- **A2.2.2.2** Describe and/or determine families of functions.

Keystone Eligible Content:
- **A2.1.2.1.1** Use exponential expressions to represent rational numbers.
- **A2.1.2.1.2** Simplify/evaluate expressions involving positive and negative exponents and/or roots (may contain all types of real numbers—exponents should not exceed power of 10).
- **A2.1.2.1.3** Simplify/evaluate expressions involving multiplying with exponents (e.g., $x^6 \cdot x^7 = x^{13}$), powers of powers (e.g., $\left(x^6\right)^7 = x^{42}$) and powers of products (e.g., $\left(2x^2\right)^3 = 8x^6$). Note: Limit to rational exponents.
- **A2.1.2.2.1** Factor algebraic expressions, including difference of squares and trinomials. Note: Trinomials limited to the form $ax^2 + bx + c$ where *a* is not equal to 0.
- **A2.1.2.2.2** Simplify rational algebraic expressions.
- **A2.1.3.1.1** Write and/or solve quadratic equations (including factoring and using the Quadratic Formula).
- **A2.2.1.1.4** Identify and/or determine the characteristics of an exponential, quadratic, or polynomial function (e.g., intervals of increase/decrease, intercepts, zeros, and asymptotes).
- **A2.2.2.1.1** Create, interpret, and/or use the equation, graph, or table of a polynomial function (including quadratics).

- **A2.2.2.1.2** Create, interpret, and/or use the equation, graph, or table of an exponential or logarithmic function (including common and natural logarithms).
- **A2.2.2.1.3** Determine, use, and/or interpret minimum and maximum values over a specified interval of a graph of a polynomial, exponential, or logarithmic function.
- **A2.2.2.1.4** Translate a polynomial, exponential, or logarithmic function from one representation of a function to another (graph, table, and equation).
- **A2.2.2.2.1** Identify or describe the effect of changing parameters within a family of functions. (e.g., $y = x^2$ and $y = x^2 + 3$, or $y = x^2$ and $y = 3x^2$).

Pennsylvania Common Core Standard(s):
- **CC.2.1.HS.F.1** Apply and extend the properties of exponents to solve problems with rational exponents.
- **CC.2.1.HS.F.3** Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs and data displays.
- **CC.2.1.HS.F.4** Use units as a way to understand problems and to guide the solution of multi-step problems.
- **CC.2.1.HS.F.7** Apply concepts of complex numbers in polynomial identities and quadratic equations to solve problems.
- **CC.2.2.HS.C.2** Graph and analyze functions and use their properties to make connections between the different representations.
- **CC.2.2.HS.C.3** Write functions or sequences that model relationships between two quantities.
- **CC.2.2.HS.C.4** Interpret the effects transformations have on functions and find the inverses of functions.
- **CC.2.2.HS.C.5** Construct and compare linear, quadratic and exponential models to solve problems.
- **CC.2.2.HS.C.6** Interpret functions in terms of the situation they model.
- **CC.2.2.HS.D.1** Interpret the structure of expressions to represent a quantity in terms of its context.
- **CC.2.2.HS.D.2** Write expressions in equivalent forms to solve problems.
- **CC.2.2.HS.D.4** Understand the relationship between zeros and factors of polynomials to make generalizations about functions and their graphs.
- **CC.2.2.HS.D.5** Use polynomial identities to solve problems.
- **CC.2.2.HS.D.7** Create and graph equations or inequalities to describe numbers or relationships.
- **CC.2.2.HS.D.9** Use reasoning to solve equations and justify the solution method.
- **CC.2.2.HS.B.2** Summarize, represent, and interpret data on two categorical and quantitative variables.

National Common Core Standard(s):
Perform arithmetic operations on polynomials.
- **CC.9-12.A.APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
Understand the relationship between zeros and factors of polynomials.
- **CC.9-12.A.APR.2** Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
- **CC.9-12.A.APR.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
Rewrite rational expressions.

- **CC.9-12.A.APR.6** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
-

Represent and solve equations and inequalities graphically.

- **CC.9-12.A.REI.11** Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Interpret the structure of expressions.

- **CC.9-12.A.SSE.2** Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $\left(x^2\right)^2 - \left(y^2\right)^2$, thus

  recognizing it as a difference of squares that can be factored as $\left(x^2 - y^2\right) - \left(x^2 + y^2\right)$.

- **CC.9-12.F.BF.3** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

Analyze functions using different representations.

- **CC.9-12.F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
  - c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- **CC.9-12.F.IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
  - a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
  - b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as $y - (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth and decay.*
- **CC.9-12.F.IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Use complex numbers in polynomial identities and equations.

- **CC.9-12.N.CN.7** Solve quadratic equations with real coefficients that have complex solutions.
- **CC.9-12.N.CN.8** Extend polynomial identities to the complex numbers. *For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.*

Solve equations and inequalities in one variable.

- **CC.9-12.A. REI.4** Solve quadratic equations in one variable.
  - a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $\left(x - p\right)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

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   b. Communicate information and ideas effectively to multiple audiences using a variety of media and formats
   d. Contribute to project teams to produce original works or solve problems
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.
   a. Plan strategies to guide inquiry
   b. Locate, organize, analyze, evaluate, synthesize, and ethically use information from a variety of sources and media
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   d. Process data and report results
4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
   a. Identify and define authentic problems and significant questions for investigation
   b. Plan and manage activities to develop a solution or complete a project
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5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.
   a. Advocate and practice safe, legal, and responsible use of information and technology
   b. Exhibit a positive attitude toward using technology that supports collaboration, learning, and productivity
   c. Demonstrate personal responsibility for lifelong learning
   d. Exhibit leadership for digital citizenship
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   a. Understand and use technology systems
   b. Select and use applications effectively and productively

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- 13.1.11.E   Justify the selection of a career.

- 13.1.11.H   Review personal high school plan against current personal career goals and select postsecondary opportunities based upon personal career interests.

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National Common Core found at www.corestandards.org
ISTE found at www.iste.org/standards/nets-for-students.aspx
Career Education and Work found at www.pacareerstandards.com/
*See Appendix for complete documents.

ELL Differentiation: Math & LA specifics found at www.pde.sas.org/module/sas/curriculumframework/elloverlay.aspx
Generic found at http://www.easad.net/esl
Todos, found at www.todos-math.org

Enrichment:
- Use polynomial identities in proofs.
- Incorporate fractions in equations.
- Graph polynomials using zeros.
- Internet/Research Activities
  - www.usatestprep.com
  - www.studyisland.com
  - www.thelearningodyssey.com
  - www.hippocampus.org
- Group/Research projects

Remediation:
- Laws of Exponents
- Scientific Notation
- Rewriting complex exponential expressions without negative exponents
- Graph quadratic functions
- Write equations of parabolas
- Solve quadratic equations by graphing
- Solve quadratic equations by factoring
- Find the number and type of solutions

IEP/GIEP: Refer to individual student's education plan under specially designed instruction.

Keystone Anchors & Eligible Content	Unit Concepts What students need to know	Unit Competencies What students need to be able to do (skills): (Students will:)	Content Vocabulary	Materials, Resources, & Instructional Activities	Assessments

A2.1.2.1 A2.1.2.1.1 A2.1.2.1.2 A2.1.2.1.3
 	
 A2.1.2.2 A2.1.2.2.1 A2.1.2.2.2
 	
 A2.1.3.1 A2.1.3.1.1
 	
 A2.2.1.1 A2.2.1.1.4
 	
 A2.2.2.1 A2.2.2.1.1 A2.2.2.1.2 A2.2.2.1.3 A2.2.2.1.4
 	
 A2.2.2.2 A2.2.2.2.1
	• Use exponents, roots, and/or absolute values to represent equivalent forms or to solve problems.
	 • Simplify expressions involving polynomials.
	 • Write and/or solve non-linear equations using various methods.
	 • Analyze and/or use patterns or relations.
	 • Create, interpret, and/or use polynomial, exponential, and/or logarithmic functions and their equations,
		• Use and apply the Properties of Exponents when problem-solving
		  o Product of Powers
		  o Power of a Power
		  o Power of a Product
		  o Negative Exponents
		  o Zero Exponent
		  o Quotient of Powers
		  o Power of a Quotient
		 • Solve problems involving scientific notation
		 • Simplify rational algebraic expressions
		 • Write and/or solve quadratic equations
		 • Identify and/or determine the characteristics of a polynomial function
		 • Classify polynomials
		  o Degree
		  o Type (e.g., Constant, Linear, Quadratic, Cubic, Quartic)
		 • Create, interpret, and/or use the equation, graph, or table of a polynomial function
			• End Behavior
			 • Monomial, Binomial, etc.
			 • Exponent
			 • Exponential Expression
			 • Rational Numbers
			 • Negative Exponents
			 • Scientific Notation
			 • Factor
			 • Factoring
			 • Difference of Squares
			 • Factor by Grouping
			 • Trinomial
			 • Multiplicity
			 • Pascal's Triangle
			 • Polynomial Function
			 • Constant
			 • Linear (Equation)
			 • Quadratic (Equation)
			 • Cubic (Polynomial)
			 • Quartic (Polynomial)
			 • Relative Maximum
			 • Maximum Value of a Graph
			 • Relative Minimum
			 • Minimum Value of a Graph
				• www.usatestprep.com
				 • www.hippocampus.org
				 • www.classzone.com
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				 • https://www.thelearningodyssey.com/
				 • Algebra 2, Concepts and Skills, (McDougal Littell, 2004)
				 • Algebra 2, (McDougal Littell, 2008)
				 • Algebra 2, Prentice Hall, 2007
				 • Common Core Algebra 2,(Pearson 2012)
				 • OnCore Mathematics, Algebra 2, (Houghton Mifflin Harcourt, 2010)
				 • McDougal Littell & Prentice Hall teaching resources
				 • Calculators
				 • CPS Clickers
				 • YouTube
				 • ESASD Valuable
					• Tests/quizzes
					 • CDTs
					 • Compass Learning
					 • Study Island
					 • USA Test Prep
					 • Observation
					 • Homework
					 • Class Work
					 • Projects
					 • PSSA Problems
					 • Journals
					 • Bell Ringers

				Video Library	
graphs, or tables. • Describe and/or determine families of functions.	o Describe end behavior • Make real-world connections while problem-solving	• Zeros • Standard Form of a Polynomial Function • Synthetic Substitution • Synthetic Division • Polynomial Long Division • Quotient Form • Repeated Solution • Turning Point • Standard Form		• New & updated resources available on departmental page	

Keystone Anchors & Eligible Content	Unit Concepts What students need to know	Unit Competencies What students need to be able to do (skills): (Students will:)	Content Vocabulary	Materials, Resources, & Instructional Activities	Assessments

A2.1.2.1
A2.1.2.1.1
A2.1.2.1.2
A2.1.2.1.3
 	
A2.1.2.2
A2.1.2.2.1
A2.1.2.2.2
 	
A2.1.3.1
A2.1.3.1.1
 	
A2.2.1.1
A2.2.1.1.4
 	
A2.2.2.1
A2.2.2.1.1
A2.2.2.1.2
A2.2.2.1.3
A2.2.2.1.4
 	
A2.2.2.2
A2.2.2.2.1
	• Use exponents, roots, and/or absolute values to represent equivalent forms or to solve problems.
	• Simplify expressions involving polynomials.
	• Write and/or solve non-linear equations using various methods.
	• Analyze and/or use patterns or relations.
	• Create, interpret, and/or use polynomial, exponential, and/or logarithmic
		• Add, subtract, and multiply polynomials
		• Use and apply special product patterns
		  o Difference of Squares
		  o Square of a Binomial
		  o Cube of a Binomial
		• Factor and solve polynomial equations
		  o Find a common monomial factor
		  o Factor the sum or difference of two cubes
		  o Factor by grouping
		  o Factor polynomials in quadratic form
		  o Use polynomial long division with and without a remainder
		  o Use synthetic division
		• Find and classify all real zeros of a polynomial function
		• Determine, use, and/or interpret minimum and maximum values, or turning
			• End Behavior
			• Monomial, Binomial, etc.
			• Exponent
			• Exponential Expression
			• Rational Numbers
			• Negative Exponents
			• Scientific Notation
			• Factor
			• Factoring
			• Difference of Squares
			• Factor by Grouping
			• Trinomial
			• Multiplicity
			• Pascal's Triangle
			• Polynomial Function
			• Constant
			• Linear (Equation)
			• Quadratic (Equation)
			• Cubic (Polynomial)
			• Quartic (Polynomial)
			• Relative Maximum
			• Maximum Value of a Graph
			• Relative Minimum
				• www.usatestprep.com
				• www.hippocampus.org
				• www.classzone.com
				• www.phsuccessnet.com
				• www.StudyIsland.com
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					• Class Work
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					• PSSA Problems
					• Journals
					• Bell Ringers

| | functions and their equations, graphs, or tables.
• Describe and/or determine families of functions. | points, over a specified interval of a graph of a polynomial function
• Solve polynomials using theorems
• Translate a polynomial, function between all representations: graph, table, and equation
• Make real-world connections while problem-solving | • Minimum Value of a Graph
• Zeros
• Standard Form of a Polynomial Function
• Synthetic Substitution
• Synthetic Division
• Polynomial Long Division
• Quotient Form
• Repeated Solution
• Turning Point
• Standard Form | • CPS Clickers
• YouTube
• ESASD Valuable Video Library
• New & updated resources available on departmental page | |

Unit 5: Radical Functions and Rational Exponents

Course Time Prior to Keystone/PSSA:
- Period – 20
- Block – 10

Overview: Students will be able to:
- Simplifying rational and Radical expression
- Solve radical equations
- Graph radical functions

Unit Essential Questions:
- Why are some solutions for radical equations extraneous?
- How do you find the domain and range of radical graphs?

Keystone Content Module/Assessment Anchor:
- **A2.1.1.1** Represent and/or use imaginary numbers in equivalent forms (e.g., square roots and exponents).
- **A2.1.1.2** Apply the order of operations in computation and in problem-solving situations.
- **A2.1.2.1** Use exponents, roots, and/or absolute values to represent equivalent forms or to solve problems.
- **A2.1.3.1** Write and/or solve non-linear equations using various methods.
- **A2.2.1.1** Analyze and/or use patterns or relations.
- **A2.2.2.2** Describe and/or determine families of functions.

Keystone Eligible Content:
- **A2.1.1.1.1** Simplify/write square roots in terms of i (e.g., $\sqrt{-24} = 2i\sqrt{6}$).
- **A2.1.2.1.1** Use exponential expressions to represent rational numbers.
- **A2.1.2.1.3** Simplify/evaluate expressions involving multiplying with exponents (e.g., $x^6 g x^7 = x^{13}$), powers of powers (e.g., $\left(x^6\right)^7 = x^{42}$) and powers of products (e.g., $\left(2x^2\right)^3 = 8x^6$). Note: Limit to rational exponents.
- **A2.1.3.1.1** Write and/or solve quadratic equations (including factoring and using the Quadratic Formula).
- **A2.1.3.1.2** Solve equations involving rational and/or radical expressions (e.g., $\dfrac{10}{(x+3)} + \dfrac{12}{(x-2)} = 1$ or $\sqrt{x} + 21x = 14$).
- **A2.2.1.1.4** Identify and/or determine the characteristics of an exponential, quadratic, or polynomial function (e.g., intervals of increase/decrease, intercepts, zeros, and asymptotes).
- **A2.2.1.1.3** Determine the domain, range, or inverse of a relation.
- **A2.2.2.2.1** Identify or describe the effect of changing parameters within a family of functions (e.g., $y = x^2$ and $y = x^2 + 3$, or $y = x^2$ and $y = 3x^2$).

Pennsylvania Common Core Standard(s):
- **CC.2.1.HS.F.1** Apply and extend the properties of exponents to solve problems with rational exponents.
- **CC.2.1.HS.F.3** Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs and data displays.
- **CC.2.1.HS.F.4** Use units as a way to understand problems and to guide the solution of multi-step problems.
- **CC.2.1.HS.F.5** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
- **CC.2.1.HS.F.6** Extend the knowledge of arithmetic operations and apply to complex numbers.
- **CC.2.1.HS.F.7** Apply concepts of complex numbers in polynomial identities and quadratic equations to solve problems.
- **CC.2.2.HS.C.1** Use the concept and notation of functions to interpret and apply them in terms of their context.
- **CC.2.2.HS.C.2** Graph and analyze functions and use their properties to make connections between the different representations.
- **CC.2.2.HS.C.3** Write functions or sequences that model relationships between two quantities.
- **CC.2.2.HS.C.4** Interpret the effects transformations have on functions and find the inverses of functions.
- **CC.2.2.HS.C.5** Construct and compare linear, quadratic and exponential models to solve problems.
- **CC.2.2.HS.C.6** Interpret functions in terms of the situation they model.
- **CC.2.2.HS.D.1** Interpret the structure of expressions to represent a quantity in terms of its context.
- **CC.2.2.HS.D.2** Write expressions in equivalent forms to solve problems.
- **CC.2.2.HS.D.4** Understand the relationship between zeros and factors of polynomials to make generalizations about functions and their graphs.
- **CC.2.2.HS.D.5** Use polynomial identities to solve problems.
- **CC.2.2.HS.D.6** Extend the knowledge of rational functions to rewrite in equivalent forms.
- **CC.2.2.HS.D.7** Create and graph equations or inequalities to describe numbers or relationships.
- **CC.2.2.HS.D.8** Apply inverse operations to solve equations or formulas for a given variable.
- **CC.2.2.HS.D.9** Use reasoning to solve equations and justify the solution method.
- **CC.2.2.HS.B.2** Summarize, represent, and interpret data on two categorical and quantitative variables.

National Common Core Standard(s):
Understand the relationship between zeros and factors of polynomials.
- **CC.9-12.A.APR.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. CC.9-12.A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Rewrite rational expressions.
- **CC.9-12.A.APR.6** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

Create equations that describe numbers or relationships.
- **CC.9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
- **CC.9-12.A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

- **CC.9-12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

Understand solving equations as a process of reasoning and explain the reasoning.

- **CC.9-12.A.REI.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Represent and solve equations and inequalities graphically.

- **CC.9-12.A.REI.11** Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f*(*x*) and *y* = *g*(*x*) intersect are the solutions of the equation *f*(*x*) = *g*(*x*); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f*(*x*) and/or *g*(*x*) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Solve equations and inequalities in one variable.

- **CC.9-12.A. REI.4** Solve quadratic equations in one variable.
  - **a.** Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x - p)^2$ = q that has the same solutions. Derive the quadratic formula from this form.
  - **b.** Solve quadratic equations by inspection (e.g., for x^2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as *a* ± *bi* for real numbers *a* and *b*.

Interpret the structure of expressions.

- **CC.9-12.A.SSE.2** Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $\left(x^2\right)^2 - \left(y^2\right)^2$, thus

  recognizing it as a difference of squares that can be factored as $\left(x^2 - y^2\right) - \left(x^2 + y^2\right)$.

Build a function that models a relationship between two quantities.

- **CC.9-12.F.BF.1** Write a function that describes a relationship between two quantities.
  - **a.** Determine an explicit expression, a recursive process, or steps for calculation from a context.
  - **b.** Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
  - **c.** Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*

Build new functions from existing functions.

- **CC.9-12.F.BF.3** Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *k f*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs.
- **CC.9-12.F.BF.4** Find inverse functions.
  - **a.** Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or f(x) = (x+1)/(x−1) for* $x \neq 1$.
  - **b.** Verify by composition that one function is the inverse of another.
  - **c.** Read values of an inverse function from a graph or a table, given that the function has an inverse.
  - **d.** Produce an invertible function from a non-invertible function by restricting the domain.

- **CC.9-12.F.BF.5** Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Extend the domain of trigonometric functions using the unit circle.
- **CC.9-12.F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x)*.
- **CC.9-12.F.IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- **CC.9-12.F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

Interpret functions that arise in applications in terms of the context.
- **CC.9-12.F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*
- **CC.9-12.F.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations.
- **CC.9-12.F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
    - a. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- **CC.9-12.F.IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
    - a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
    - b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as $y - (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth and decay.*
- **CC.9-12.F.IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Perform arithmetic operations with complex numbers.
- **CC.9-12.N.CN.1** Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form *a + bi* with *a* and *b* real.
- **CC.9-12.N.CN.2** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Use complex numbers in polynomial identities and equations.
- **CC.9-12.N.CN.7** Solve quadratic equations with real coefficients that have complex solutions.

- **CC.9-12.N.CN.8** Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* (x + 2i)(x – 2i).

ISTE Standards:
1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
    b. Create original works as a means of personal or group expression
    c. Use models and simulations to explore complex systems and issues
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
    a. Interact, collaborate, and publish with peers, experts, or others employing a variety of digital environments and media
    b. Communicate information and ideas effectively to multiple audiences using a variety of media and formats
    d. Contribute to project teams to produce original works or solve problems
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.
    a. Plan strategies to guide inquiry
    b. Locate, organize, analyze, evaluate, synthesize, and ethically use information from a variety of sources and media
    c. Evaluate and select information sources and digital tools based on the appropriateness to specific tasks
    d. Process data and report results
4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
    a. Identify and define authentic problems and significant questions for investigation
    b. Plan and manage activities to develop a solution or complete a project
    c. Collect and analyze data to identify solutions and/or make informed decisions
    d. Use multiple processes and diverse perspectives to explore alternative solutions
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.
    a. Advocate and practice safe, legal, and responsible use of information and technology
    b. Exhibit a positive attitude toward using technology that supports collaboration, learning, and productivity
    c. Demonstrate personal responsibility for lifelong learning
    d. Exhibit leadership for digital citizenship
6. Technology Operations and Concepts – Students demonstrate a sound understanding of technology concepts, systems, and operations.
    a. Understand and use technology systems
    b. Select and use applications effectively and productively

Career Education and Work Standards:
- 13.1.11.A   Relate careers to individual interests, abilities, and aptitudes.
- 13.1.11.E   Justify the selection of a career.
- 13.1.11.H   Review personal high school plan against current personal career goals and select postsecondary opportunities based upon personal career interests.

Connecting to Common Core and Other Standards:
PA Standards found at www.pdesas.org/standards/standardsdownloads
National Common Core found at www.corestandards.org
ISTE found at www.iste.org/standards/nets-for-students.aspx
Career Education and Work found at www.pacareerstandards.com/
*See Appendix for complete documents.

ELL Differentiation: Math & LA specifics found at www.pde.sas.org/module/sas/curriculumframework/elloverlay.aspx
Generic found at http://www.easad.net/esl
Todos, found at www.todos-math.org

Enrichment:
- Using step-function to round square roots
- Simplifying radicals within a radical
- Solving more complicated radical equations
- Transformations of radical functions
- Internet/Research Activities
  - www.usatestprep.com
  - www.studyisland.com
  - www.thelearningodyssey.com
  - www.hippocampus.org
- Group/Research projects

Remediation:
- Simplifying radicals
- Rewriting simple rational exponents to radical and vice versa
- Solving linear and quadratic equations

IEP/GIEP: Refer to individual student's education plan under specially designed instruction.

Keystone Anchors & Eligible Content	Unit Concepts What students need to know	Unit Competencies What students need to be able to do (skills): (Students will:)	Content Vocabulary	Materials, Resources, & Instructional Activities	Assessments

A2.1.1.1
A2.1.1.1.1
 	
A2.1.1.2
 	
A2.1.2.1
A2.1.2.1.1
A2.1.2.1.3
 	
A2.1.3.1
A2.1.3.1.1
A2.1.3.1.2
 	
A2.2.1.1
A2.2.1.1.3
A2.2.1.1.4
 	
A2.2.2.2
A2.2.2.2.1
	• Represent and/or use imaginary numbers in equivalent forms (e.g., square roots and exponents).
	• Apply the order of operations in computation and in problem-solving situations.
	• Use exponents, roots, and/or absolute values to represent equivalent forms or to solve problems.
	• Write and/or solve non-linear equations using various methods.
	• Analyze and/or use patterns or relations.
	• Describe and/or determine families of
		• Simplify nth roots and radical expressions
		• Multiply and divide radicals
		• Add and subtract radical expressions
		• Multiply and divide binomial radical expressions
		• Simplify and evaluate expressions with rational exponents
		• Convert to and from radical form
		• Simplify numbers with rational exponents
		  ○ Without a calculator
		  ○ With a calculator
		• Solve square root equations
		• Solve radical equations with rational exponents
		• Make real-world connections while problem-solving
			• Radicals
			• Like Radicals
			• nth Root
			• Index of a Radical
			• Principal Root
			• Radical Equation
			• Radical Function
			• Radicand
			• Rational Numbers
			• Rational Exponents
			• Rationalizing the Denominator
			• Square Root Equation
			• Square Root Function
			• Imaginary Numbers
			• Absolute Values
			• Equivalent Forms
			• Patterns
			• Functions
			• Relations
			• Simplest Form
			• Inverse Relation
			• Inverse Function
			• Power Function
			• Cubic Function
			• Horizontal Line Test
				• www.usatestprep.com
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			Domain
			Range
functions.			Extraneous Solution

Keystone Anchors & Eligible Content	Unit Concepts What students need to know	Unit Competencies What students need to be able to do (skills): (Students will:)	Content Vocabulary	Materials, Resources, & Instructional Activities	Assessments

A2.1.1.1
A2.1.1.1.1
 	
A2.1.1.2
 	
A2.1.2.1
A2.1.2.1.1
A2.1.2.1.3
 	
A2.1.3.1
A2.1.3.1.1
A2.1.3.1.2
 	
A2.2.1.1
A2.2.1.1.3
A2.2.1.1.4
 	
A2.2.2.2
A2.2.2.2.1
	• Describe and/or determine families of functions
	• Multiplying and dividing radical expressions
	• Roots and radical expressions
	• Binomial radical expressions
	• Rational Exponents
	• Solving square roots and other radical equations
	• Functions operations
	• Inverse relations and functions
	• Graphing Square roots and other radical functions
		• Solve equations with two rational exponents
		• Check for extraneous solutions
		• Add, subtract, multiply and divide functions
		• Compose functions
		• Find and verify the inverse of a
		  ○ Function
		  ○ Power function
		  ○ Cubic Function
		• Graph a relation and its inverse
		• Compose inverse functions
		• Solve square and cube root functions by graphing
		• Translate and graph radical functions vertically and horizontally
		• Solve radical equations and/or equations with two radicals
		• Solve a radical equation when given a function
			• Radicals
			• Like Radicals
			• nth Root
			• Index of a Radical
			• Principal Root
			• Radical Equation
			• Radical Function
			• Radicand
			• Rational Numbers
			• Rational Exponents
			• Rationalizing the Denominator
			• Square Root Equation
			• Square Root Function
			• Imaginary Numbers
			• Absolute Values
			• Equivalent Forms
			• Patterns
			• Functions
			• Relations
			• Simplest Form
			• Inverse Relation
			• Inverse Function
			• Power Function
			• Cubic Function
				• www.usatestprep.com
				• www.hippocampus.org
				• www.classzone.com
				• www.phsuccessnet.com
				• www.StudyIsland.com
				• https://www.thelearningodyssey.com/
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					• Compass Learning
					• Study Island
					• USA Test Prep
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					• Homework
					• Class Work
					• Projects
					• PSSA Problems
					• Journals
					• Bell Ringers

		- Solve an equation with an extraneous solution - Make real-world connections while problem-solving	- Horizontal Line Test - Domain - Range - Extraneous Solution	

Unit 6: Exponential and Logarithmic Functions

Course Time Prior to Keystone/PSSA:
- Period – 24
- Block – 12

Overview: Students will be able to:
- Graph exponential and logarithmic functions
- Simplify exponential and logarithmic expressions
- Solve exponential and logarithmic equations
- Apply exponential and logarithmic equations to the real-world

Unit Essential Questions:
- What is an exponential function?
- What is a logarithmic function?
- What is the relationship of exponential functions to logarithmic functions?

Keystone Content Module/Assessment Anchor:
- **A2.1.1.1** Represent and/or use imaginary numbers in equivalent forms (e.g., square roots and exponents).
- **A2.1.1.2** Apply the order of operations in computation and in problem-solving situations
- **A2.1.2.1** Use exponents, roots, and/or absolute values to represent equivalent forms or to solve problems.
- **A2.1.2.2** Simplify expressions involving polynomials.
- **A2.1.3.1** Write and/or solve non-linear equations using various methods.
- **A2.2.1.1** Analyze and/or use patterns or relations.
- **A2.2.2.1** Create, interpret, and/or use polynomial, exponential, and/or logarithmic functions and their equations, graphs, or tables.
- **A2.2.2.2** Describe and/or determine families of functions.
- **A2.2.3.1** Analyze and/or interpret data on a scatter plot to make predictions.

Keystone Eligible Content:
- **A2.1.2.1.1** Use exponential expressions to represent rational numbers.
- **A2.1.2.1.2** Simplify/evaluate expressions involving positive and negative exponents and/or roots (may contain all types of real numbers—exponents should not exceed power of 10).
- **A2.1.2.1.3** Simplify/evaluate expressions involving multiplying with exponents (e.g., $x^6 \text{g} x^7 = x^{13}$), powers of powers (e.g., $\left(x^6\right)^7 = x^{42}$) and powers of products (e.g., $\left(2x^2\right)^3 = 8x^6$). Note: Limit to rational exponents.
- **A2.1.2.1.4** Simplify or evaluate expressions involving logarithms and exponents (e. g., $\log_2 8 = 3$ or $\log_4 2 = \dfrac{1}{2}$)

- **A2.1.2.2.1** Factor algebraic expressions, including difference of squares and trinomials. Note: Trinomials limited to the form $ax^2 + bx + c$ where a is not equal to 0.
- **A2.1.2.2.2** Simplify rational algebraic expressions.
- **A2.1.3.1.1** Write and/or solve quadratic equations (including factoring and using the Quadratic Formula).
- **A2.1.3.1.2** Solve equations involving rational and/or radical expressions (e.g., $\dfrac{10}{(x+3)} + \dfrac{12}{(x-2)} = 1$ or $\sqrt{x} + 21x = 14$).
- **A2.1.3.1.3** Write and/or solve a simple exponential or logarithmic equation (including common and natural logarithms).
- **A2.1.3.1.4** Write, solve, and/or apply linear or exponential growth or decay (including problem situations).
- **A2.2.1.1.1** Analyze a set of data for the existence of a pattern and represent the pattern with a rule algebraically and/or graphically.
- **A2.2.1.1.2** Identify and/or extend a pattern as either an arithmetic or geometric sequence (e. g., given a geometric sequence, find the 20th term).
- **A2.2.1.1.3** Determine the domain, range, or inverse of a relation.
- **A2.2.1.1.4** Identify and/or determine the characteristics of an exponential, quadratic, or polynomial function (e.g., intervals of increasing/decreasing, intercepts, zeros, and asymptotes).
- **A2.2.2.1.1** Create, interpret, and/or use the equation, graph, or table of a polynomial function (including quadratics).
- **A2.2.2.1.2** Create, interpret, and/or use the equation, graph, or table of an exponential or logarithmic function (including common and natural logarithms).
- **A2.2.2.1.3** Determine, use, and/or interpret minimum and maximum values over a specified interval of a graph of a polynomial, exponential, or logarithmic function.
- **A2.2.2.1.4** Translate a polynomial, exponential, or logarithmic function from one representation of a function to another (graph, table, and equation).
- **A2.2.2.2.1** Identify or describe the effect of changing parameters within a family of functions (e.g., $y = x^2$ and $y = x^2 + 3$, or $y = x^2$ and $y = 3x^2$).
- **A2.2.3.1.1** Draw, identify, find, interpret and/or write an equation for a regression model (lines and curves of best fit) for a scatter plot.
- **A2.2.3.1.2** Make predictions using the equations or graphs of regression models (lines and curves of best fit) of scatter plot.

Pennsylvania Common Core Standard(s):
- **CC.2.1.HS.F.1** Apply and extend the properties of exponents to solve problems with rational exponents.
- **CC.2.1.HS.F.3** Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs and data displays.
- **CC.2.1.HS.F.4** Use units as a way to understand problems and to guide the solution of multi-step problems.
- **CC.2.1.HS.F.5** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
- **CC.2.1.HS.F.6** Extend the knowledge of arithmetic operations and apply to complex numbers.
- **CC.2.1.HS.F.7** Apply concepts of complex numbers in polynomial identities and quadratic equations to solve problems.
- **CC.2.2.HS.D.1** Interpret the structure of expressions to represent a quantity in terms of its context.
- **CC.2.2.HS.D.2** Write expressions in equivalent forms to solve problems.
- **CC.2.2.HS.D.4** Understand the relationship between zeros and factors of polynomials to make generalizations about functions and their graphs.

- **CC.2.2.HS.D.5** Use polynomial identities to solve problems.
- **CC.2.2.HS.D.6** Extend the knowledge of rational functions to rewrite in equivalent forms.
- **CC.2.2.HS.D.7** Create and graph equations or inequalities to describe numbers or relationships.
- **CC.2.2.HS.D.8** Apply inverse operations to solve equations or formulas for a given variable.
- **CC.2.2.HS.D.9** Use reasoning to solve equations and justify the solution method.
- **CC.2.2.HS.D.10** Represent, solve and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.
- **CC.2.2.HS.C.2** Graph and analyze functions and use their properties to make connections between the different representations.
- **CC.2.2.HS.C.3** Write functions or sequences that model relationships between two quantities.
- **CC.2.2.HS.C.4** Interpret the effects transformations have on functions and find the inverses of functions.
- **CC.2.2.HS.C.5** Construct and compare linear, quadratic and exponential models to solve problems.
- **CC.2.2.HS.C.6** Interpret functions in terms of the situation they model.
- **CC.2.2.HS.C.9** Use reasoning to solve equations and justify the solution method.
- **CC.2.2.HS.B.2** Summarize, represent, and interpret data on two categorical and quantitative variables.

National Common Core Standard(s):
Understand the concept of function and use function notation.
- **CC.9-12.F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x)*.
- **CC.9-12.F.IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Interpret functions that arise in application in terms of the context.
- **CC.9-12.F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
- **CC.9-12.F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*
- **CC.9-12.F.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations.
- **CC.9-12.F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
  - **b.** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions
- **CC.9-12.F.IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
  - **a.** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of the context.

   b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y – (1.02)^t, y = (0.97)^t, y = (1.01)^{12t}, y = (1.2)$^{t/10}$, and classify them as representing exponential growth and decay.*

- **CC.9-12.F.IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Build new functions from existing functions.
- **CC.9-12.F.BF.3** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs.

Write expressions in equivalent forms to solve problems.
- **CC.9-12.A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

   c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be*

rewritten as $\left(1.15^{1/12}\right)^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

- **CC.9-12.A.SSE 4**. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*

Create equations that describe numbers or relationships.
- **CC.9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
- **CC.9-12.A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **CC.9-12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

Interpret the structure of expressions.
- **CC.9-12.A.SSE.2** Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $\left(x^2\right)^2 - \left(y^2\right)^2$, thus

recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Perform arithmetic operations with complex numbers.
- **CC.9-12.N.CN.1** Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
- **CC.9-12.N.CN.2** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Use complex numbers in polynomial identities and equations.
- **CC.9-12.N.CN.7** Solve quadratic equations with real coefficients that have complex solutions.
- **CC.9-12.N.CN.8** Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as (x + 2i)(x – 2i).*

Understand the relationship between zeros and factors of polynomials.

- **CC.9-12.A.APR.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Rewrite rational expressions.

- **CC.9-12.A.APR.6** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

Represent and solve equations and inequalities graphically.

- **CC.9-12.A.REI.10.** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- **CC.9-12.A.REI.11** Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Solve equations and inequalities in one variable.

- **CC.9-12.A. REI.4** Solve quadratic equations in one variable.

    a. Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

    b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b*.

Construct and compare linear, quadratic, and exponential models and solve problems.

- **CC.9-12.F.LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions.

    a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

    b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

    c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

- **CC.9-12.F.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- **CC.9-12.F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
- **CC.9-12.F.LE.4** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.

ISTE Standards:

1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.

    b. Create original works as a means of personal or group expression

     c. Use models and simulations to explore complex systems and issues

2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
    a. Interact, collaborate, and publish with peers, experts, or others employing a variety of digital environments and media
    b. Communicate information and ideas effectively to multiple audiences using a variety of media and formats
    d. Contribute to project teams to produce original works or solve problems
3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.
    a. Plan strategies to guide inquiry
    b. Locate, organize, analyze, evaluate, synthesize, and ethically use information from a variety of sources and media
    c. Evaluate and select information sources and digital tools based on the appropriateness to specific tasks
    d. Process data and report results
4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
    a. Identify and define authentic problems and significant questions for investigation
    b. Plan and manage activities to develop a solution or complete a project
    c. Collect and analyze data to identify solutions and/or make informed decisions
    d. Use multiple processes and diverse perspectives to explore alternative solutions
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.
    a. Advocate and practice safe, legal, and responsible use of information and technology
    b. Exhibit a positive attitude toward using technology that supports collaboration, learning, and productivity
    c. Demonstrate personal responsibility for lifelong learning
    d. Exhibit leadership for digital citizenship
6. Technology Operations and Concepts – Students demonstrate a sound understanding of technology concepts, systems, and operations.
    a. Understand and use technology systems
    b. Select and use applications effectively and productively

Career Education and Work Standards:
- 13.1.11.A   Relate careers to individual interests, abilities, and aptitudes.
- 13.1.11.E   Justify the selection of a career.
- 13.1.11.H   Review personal high school plan against current personal career goals and select postsecondary opportunities based upon personal career interests.

Connecting to Common Core and Other Standards:
    PA Standards found at www.pdesas.org/standards/standardsdownloads
    National Common Core found at www.corestandards.org
    ISTE found at www.iste.org/standards/nets-for-students.aspx
    Career Education and Work found at www.pacareerstandards.com/
    *See Appendix for complete documents.

ELL Differentiation: Math & LA specifics found at www.pde.sas.org/module/sas/curriculumframework/elloverlay.aspx

Generic found at http://www.easad.net/esl
Todos, found at www.todos-math.org

Enrichment:
- Calculating retirement using exponential equations
- Using the logarithms to calculate the intensity of an earthquake
- Proving logarithmic properties
- Calculating the natural base
- Solving logarithmic equations
- Internet/Research Activities
    - www.usatestprep.com
    - www.studyisland.com
    - www.thelearningodyssey.com
    - www.hippocampus.org
- Group/Research projects

Remediation:
- Solve simple exponential equations
- Estimate powers of bases
- Solving quadratic equations
- Solving radical equations
- Transformation from exponential to logarithm and vice versa

IEP/GIEP: Refer to individual student's education plan under specially designed instruction.

Keystone Anchors & Eligible Content	Unit Concepts What students need to know	Unit Competencies What students need to be able to do (skills): (Students will:)	Content Vocabulary	Materials, Resources, & Instructional Activities	Assessments

A2.1.1.1
 	
A2.1.1.2
 	
A2.1.2.1
A2.1.2.1.1
A2.1.2.1.2
A2.1.2.1.3
A2.1.2.1.4
 	
A2.1.2.2
A2.1.2.2.1
A2.1.2.2.2
 	
A2.1.3.1
A2.1.3.1.1
A2.1.3.1.2
A2.1.3.1.3
A2.1.3.1.4
 	
A2.2.1.1
A2.2.1.1.1
A2.2.1.1.2
A2.2.1.1.3
A2.2.1.1.4
 	
A2.2.2.1
A2.2.2.1.1
A2.2.2.1.2
A2.2.2.1.3
	• Represent and/or use imaginary numbers in equivalent forms (e.g., square roots and exponents).
	• Apply the order of operations in computation and in problem-solving situations
	• Use exponents, roots, and/or absolute values to represent equivalent forms or to solve problems.
	• Simplify expressions involving polynomials.
	• Write and/or solve non-linear equations using various
		• Identify the role of constants in $y = ab^{cx}$
		• Graph and solve problems using the growth factor and exponential growth functions
		• Identify Asymptotes
		• Identify parent functions for exponential growth and decay functions
		• Find the domain and range for exponential functions
		• Solve compound interest problems
		• Graph and solve problems using the decay factor and exponential decay functions
		• Solve continually compounded interest problems
		• Use e as a base to evaluate and/or graph exponential functions
		• Write, graph and/or evaluate exponential and logarithmic functions
		• Use properties of logarithms
		• Evaluate logarithms
		• Graph logarithmic functions
		• Solve exponential and logarithmic equations by
		  o Equating exponents
		  o Taking a logarithm of each side
			• Asymptote
			• Change of Base Formula
			• Exponential Equation
			• Exponential Growth
			• Growth Factor
			• Exponential Decay
			• Domain
			• Range
			• Natural Base e or *Euler Number*
			• Compound Interest
			• Continuously Compounded Interest Formula
			• Decay Factor
			• Common Logarithm
			• Natural Logarithm
			• Logarithm
			• Logarithmic Equation
				• www.usatestprep.com
				• www.hippocampus.org
				• www.classzone.com
				• www.phsuccessnet.com
				• www.StudyIsland.com
				• https://www.thelearningodyssey.com/
				• Algebra 2, Concepts and Skills, (McDougal Littell, 2004)
				• Algebra 2, (McDougal Littell, 2008)
				• Algebra 2, Prentice Hall, 2007
				• Common Core Algebra 2,(Pearson 2012)
				• OnCore Mathematics, Algebra 2, (Houghton Mifflin Harcourt, 2010)
				• McDougal Littell & Prentice Hall teaching resources
				• Calculators
				• CPS Clickers
				• YouTube
				• ESASD Valuable Video Library
				• New & updated resources available on departmental
					• Tests/quizzes
					• CDTs
					• Compass Learning
					• Study Island
					• USA Test Prep
					• Observation
					• Homework
					• Class Work
					• Projects
					• PSSA Problems
					• Journals
					• Bell Ringers

A2.2.2.1.4 A2.2.2.2 A2.2.2.2.1	methods. • Analyze and/or use patterns or relations.	o Using an exponential model o Solving a logarithmic equation o Exponentiating each side of an equation • Rewrite logarithmic equations into exponential form • Make real-world connections while problem-solving	• Logarithmic Function • Natural Logarithmic Function • Regression • Scatter plot	page	
Keystone Anchors & Eligible Content	**Unit Concepts** What students need to know	**Unit Competencies** What students need to be able to do (skills): (Students will:)	**Content Vocabulary**	**Materials, Resources, & Instructional Activities**	**Assessments**
A2.2.3.1 A2.2.3.1.1 A2.2.3.1.2	• Describe and/or determine families of functions. • Create, interpret, and/or use polynomial, exponential, and/or logarithmic functions and their equations, graphs, or tables. • Analyze and/or interpret data on a scatter plot to make predictions.	• Find the equation for the line of best fit for a scatter plot o Using Algebra I skills learned o Using a graphing calculator • Make exponential and logarithmic real-world connections o Find an exponential model from a set of data or scatter plot o Use exponential regression on a graphing calculator o Write a power function when give two points from its graph o Find a power model from a set of data or scatter plot o Use power regression on a graphing calculator o Use the correlation coefficient to determine which equation fits the data best – exponential or logarithmic	• Asymptote • Change of Base Formula • Exponential Equation • Exponential Growth • Growth Factor • Exponential Decay • Domain • Range • Natural Base *e* or *Euler Number* • Compound Interest • Continuously Compounded Interest Formula • Decay Factor • Common Logarithm • Natural Logarithm • Logarithm • Logarithmic	• www.usatestprep.com • www.hippocampus.org • www.classzone.com • www.phsuccessnet.com • www.StudyIsland.com • https://www.thelearningod yssey.com/ • Algebra 2, Concepts and Skills, (McDougal Littell, 2004) • Algebra 2, (McDougal Littell, 2008) • Algebra 2, Prentice Hall, 2007 • Common Core Algebra 2,(Pearson 2012) • OnCore Mathematics, Algebra 2, (Houghton Mifflin Harcourt, 2010) • McDougal Littell & Prentice Hall teaching resources • Calculators • CPS Clickers • YouTube • ESASD Valuable Video Library • New & updated resources	• Tests/quizzes • CDTs • Compass Learning • Study Island • USA Test Prep • Observation • Homework • Class Work • Projects • PSSA Problems • Journals • Bell Ringers

			Equation • Logarithmic Function • Natural Logarithmic Function • Regression • Scatter plot	available on departmental page	

Unit 7: Rational Functions and Probability

Course Time Prior to Keystone/PSSA:
- Period – 24
- Block – 12

Overview: Students will be able to:
- Solve direct, inverse, and joint variation problems
- Simplify rational expressions by adding, subtracting, multiplying and dividing
- Solve and graph rational equations and functions
- Find the probability of single, compound, independent, and dependent events
- Use combination, permutation and the counting principle to solve problems involving probability

Unit Essential Questions:
- What are rational expressions?
- How is a rational function solved?
- What role does an undefined value play in the graph of a rational function?
- How is probability used to solve problems?

Keystone Content Module/Assessment Anchor:
- **A2.1.2.2** Simplify expressions involving polynomials.
- **A2.1.3.1** Write and/or solve non-linear equations using various methods.
- **A2.1.3.2** Describe and/or determine change.
- **A2.2.1.1** Analyze and/or use patterns or relations.
- **A2.2.2.2** Describe and/or determine families of functions.
- **A2.2.3.2** Apply probability to practical situations.

Keystone Eligible Content:
- **A2.1.2.2.1** Factor algebraic expressions, including difference of squares and trinomials. Note: Trinomials limited to the form ax_2+bx+c where a is not equal to 0.
- **A2.1.2.2.2** Simplify rational algebraic expressions
- **A2.1.3.1.1** Write and/or solve quadratic equations (including factoring and using the Quadratic Formula).
- **A2.1.3.1.2** Determine how a change in one variable relates to change in a second variable (e. g. y=4/x; if x doubles what happens to y?)
- **A2.1.3.1.2** Solve equations involving rational and/or radical expressions (e.g., $10/(x + 3) + 12/(x − 2) = 1$ or _x_ + 21x = 14).
- **A2.2.1.1.3** Determine the domain, range, or inverse of a relation.
- **A2.2.1.1.4** Identify and/or determine the characteristics of an exponential, quadratic, or polynomial function (e.g., intervals of increasing/decreasing, intercepts, zeros, and asymptotes).

- **A2.2.2.2.1** Identify or describe the effect of changing parameters within a family of functions (e.g., $y = x^2$ and $y = x^2 + 3$, or $y = x^2$ and $y = 3x^2$).
- **A2.2.3.2.1** Use combinations, permutation and the fundamental counting principle to solve problems involving probability.
- **A2.2.3.2.2** Use odds to find the probability and/or use probability to find odds.
- **A2.2.3.2.3** Use probability for independent, dependent, or compound events to predict outcomes.

Pennsylvania Common Core Standard(s):
- **CC.2.2.HS.D.1** Interpret the structure of expressions to represent a quantity in terms of its context.
- **CC.2.2.HS.D.2** Write expressions in equivalent forms to solve problems.
- **CC.2.2.HS.D.3** Extend knowledge of arithmetic operations and apply to polynomials.
- **CC.2.2.HS.D.4** Understand the relationship between zeros and factors of polynomials to make generalizations about functions and their graphs.
- **CC.2.2.HS.D.5** Use polynomial identities to solve problems.
- **CC.2.2.HS.D.6** Extend the knowledge of rational functions to rewrite in equivalent forms.
- **CC.2.2.HS.D.7** Create and graph equations or inequalities to describe numbers or relationships.
- **CC.2.2.HS.D.8** Apply inverse operations to solve equations or formulas for a given variable.
- **CC.2.2.HS.D.9** Use reasoning to solve equations and justify the solution method.
- **CC.2.2.HS.D.10** Represent, solve and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.
- **CC.2.2.HS.C.2** Graph and analyze functions and use their properties to make connections between the different representations.
- **CC.2.2.HS.C.3** Write functions or sequences that model relationships between two quantities.
- **CC.2.2.HS.C.4** Interpret the effects transformations have on functions and find the inverses of functions.
- **CC.2.2.HS.C.6** Interpret functions in terms of the situation they model.
- **CC.2.2.HS.C.9** Use reasoning to solve equations and justify the solution method.
- **CC.2.2.HS.B.2** Summarize, represent, and interpret data on two categorical and quantitative variables.
- **CC.2.2.HS.B.6** Use concepts of independence and conditional probability to interpret data.
- **CC.2.2.HS.B.7** Apply the rules of probability to compute probabilities of compound events in a uniform probability model.

National Common Core Standard(s):
Understand the relationship between zeros and factors of polynomials
- **CC.9-12.A.APR.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
Rewrite rational expressions
- **CC.9-12.A.APR.6** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
- **CC.9-12.A.APR.7** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
Create equations that describe numbers or relationships

- **CC.9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
- **CC.9-12.A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V =IR to highlight resistance R.*

Understand solving equations as a process of reasoning and explain the reasoning
- **CC.9-12.A. REI.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve equations and inequalities in one variable
- **CC.9-12.A. REI.4** B. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Represent and solve equations and inequalities graphically
- **CC.9-12.A.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate
- **CC.9-12.A.REI.11** Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f*(*x*) and *y* = *g*(*x*) intersect are the solutions of the equation *f*(*x*) = *g*(*x*); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f*(*x*) and/or *g*(*x*) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Interpret the structure of expressions
- **CC.9-12.A.SSE.1** Interpret expressions that represent a quantity in terms of its context.

- **CC.9-12.A.SSE.2** Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $\left(x^2\right)^2 - \left(y^2\right)^2$, thus

  recognizing it as a difference of squares that can be factored as $\left(x^2 - y^2\right) - \left(x^2 + y^2\right)$.

Write expressions in equivalent forms to solve problems
- **CC.9-12.A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
    - a. Factor a quadratic expression to reveal the zeros of the function it defines.

Build new functions from existing functions
- **CC.9-12.F.BF.3** Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *k f*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
- **CC.9-12.F.BF.4** Find inverse functions.
    - a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) = 2x^3$ or f(x) = (x+1)/(x−1) for $x \neq 1$.*
    - b. Verify by composition that one function is the inverse of another.
    - c. Read values of an inverse function from a graph or a table, given that the function has an inverse.
    - d. Produce an invertible function from a non-invertible function by restricting the domain.

Understand the concept of a function and use function notation

- **CC.9-12.F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x)*.
- **CC.9-12.F.IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Interpret functions that arise in applications in terms of the context
- **CC.9-12.F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
- **CC.9-12.F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

Analyze functions using different representations
- **CC.9-12.F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
    - d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
- **CC.9-12.F.IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
    - a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Use the rules of probability to compute probabilities of compound events in a uniform probability model
- **CC.9-12.S.CP.6** Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A,* and interpret the answer in terms of the model.
- **CC.9-12.S.CP.7** Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.
- **CC.9-12.S.CP.8** Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.
- **CC.9-12.S.CP.9** Use permutations and combinations to compute probabilities of compound events and solve problems.

ISTE Standards:
1. Creativity and Innovation – Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.
    - b. Create original works as a means of personal or group expression
    - c. Use models and simulations to explore complex systems and issues
2. Communication and Collaboration – Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
    - a. Interact, collaborate, and publish with peers, experts, or others employing a variety of digital environments and media
    - b. Communicate information and ideas effectively to multiple audiences using a variety of media and formats
    - d. Contribute to project teams to produce original works or solve problems

3. Research and Information Fluency – Students apply digital tools to gather, evaluate, and use information.
   a. Plan strategies to guide inquiry
   b. Locate, organize, analyze, evaluate, synthesize, and ethically use information from a variety of sources and media
   c. Evaluate and select information sources and digital tools based on the appropriateness to specific tasks
   d. Process data and report results

4. Critical Thinking, Problem Solving, and Decision Making – Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
   a. Identify and define authentic problems and significant questions for investigation
   b. Plan and manage activities to develop a solution or complete a project
   c. Collect and analyze data to identify solutions and/or make informed decisions
   d. Use multiple processes and diverse perspectives to explore alternative solutions
5. Digital Citizenship – Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.
   a. Advocate and practice safe, legal, and responsible use of information and technology
   b. Exhibit a positive attitude toward using technology that supports collaboration, learning, and productivity
   c. Demonstrate personal responsibility for lifelong learning
   d. Exhibit leadership for digital citizenship
6. Technology Operations and Concepts – Students demonstrate a sound understanding of technology concepts, systems, and operations.
   a. Understand and use technology systems
   b. Select and use applications effectively and productively

Career Education and Work Standards:
- 13.1.11.A   Relate careers to individual interests, abilities, and aptitudes.
- 13.1.11.E   Justify the selection of a career.
- 13.1.11.H   Review personal high school plan against current personal career goals and select postsecondary opportunities based upon personal career interests.

Connecting to Common Core and Other Standards:
PA Standards found at www.pdesas.org/standards/standardsdownloads
National Common Core found at www.corestandards.org
ISTE found at www.iste.org/standards/nets-for-students.aspx
Career Education and Work found at www.pacareerstandards.com/
*See Appendix for complete documents.

ELL Differentiation: Math & LA specifics found at www.pde.sas.org/module/sas/curriculumframework/elloverlay.aspx
Generic found at http://www.easad.net/esl
Todos, found at www.todos-math.org

Enrichment:
- Finding probabilities of multiples
- Proportions use in sciences
- Using direct and indirect variation to solve real-world problems
- Internet/Research Activities
    - www.usatestprep.com
    - www.studyisland.com
    - www.thelearningodyssey.com
    - www.hippocampus.org
- Group/Research projects

Remediation:
- Factoring polynomials
- Solving polynomial equations
- Solving proportions
- Direct variations
- Finding theoretical probability

IEP/GIEP: Refer to individual student's education plan under specially designed instruction.

Keystone Anchors & Eligible Content	Unit Concepts What students need to know	Unit Competencies What students need to be able to do (skills): (Students will:)	Content Vocabulary	Materials, Resources, & Instructional Activities	Assessments
A2.1.2.2 A2.1.2.2.1 A2.1.2.2.2 A2.1.3.1 A2.1.3.1.1 A2.1.3.1.2 A2.1.3.1.3 A2.1.3.2 A2.2.1.1 A2.2.1.1.4 A2.2.2.2 A2.2.2.2.1 A2.2.3.2 A2.2.3.2.1 A2.2.3.2.2 A2.2.3.2.3	• Simplify expressions involving polynomials. • Write and/or solve non-linear equations using various methods. • Describe and/or determine change. • Analyze and/or use patterns or relations. • Describe and/or determine families of functions. • Apply probability to practical situations. • The reciprocal function family • Inverse variation • Rational functions and their graphs • Adding and	• Classify and/or identify direct and inverse variation • Write and model inverse variation equations • Use data to identify inverse variation • Write a joint variation equation • Compare different types of variation • Graph rational functions • Simplify rational expressions using addition, subtraction, multiplication, and division • Solve rational equations and functions • Identify undefined values of rational functions • Define the domain of a rational function • Graph rational functions by identifying the asymptotes • Apply the Fundamental Counting Principle • Find the probability of single events • Find probability of compound events • Solve problems with independent and dependent	• Asymptotes • Branch • Complex Fraction • Inverse Variation • Joint Variation • Constant of Variation • Mutually Exclusive Events • Point Discontinuity • Rational Function • Reciprocal Function • Simplest Form • Complex Fraction • Cross Multiplication • Probability • Theoretical Probability • Experimental Probability • Geometric Probability • Fundamental Counting Principle • Tree Diagram • Permutation • Factorial • Combination • Binomial Theorem	• www.usatestprep.com • www.hippocampus.org • www.classzone.com • www.phsuccessnet.com • www.StudyIsland.com • https://www.thelearningodyssey.com/ • Algebra 2, Concepts and Skills, (McDougal Littell, 2004) • Algebra 2, (McDougal Littell, 2008) • Algebra 2, Prentice Hall, 2007 • Common Core Algebra 2,(Pearson 2012) • OnCore Mathematics, Algebra 2, (Houghton Mifflin Harcourt, 2010) • McDougal Littell & Prentice Hall teaching resources • Graphing Calculators • CPS Clickers • YouTube • ESASD Valuable Video Library • New & updated resources available on departmental page	• Tests/quizzes • CDTs • Compass Learning • Study Island • USA Test Prep • Observation • Homework • Class Work • Projects • PSSA Problems • Journals • Bell Ringers

	subtracting rational expressions • Solving rational equations	events • Use combinations and permutations to solve problems • Make a real-world connections while solving problems	• Pascal's Triangle • Dependent Events • Independent Events • Compound Event • Overlapping Events • Disjoint or Mutually Exclusive Events • Venn Diagram		