2.5 Multiplying Real Numbers

Goal

Multiply real numbers using the rule for the sign of a product.

Key Words

- closure property
- commutative property
- associative property
- identity property
- property of zero
- property of negative one

How far did a flying squirrel drop?

An object's change in position when it drops can be found by multiplying its velocity by the time it drops. In Example 4 you will find the change in position of a squirrel.

The product of any two real numbers is itself a unique real number. We say that the real numbers are *closed* under multiplication. This fact is called the **closure property of real number multiplication**. Multiplication by a positive integer can be modeled as repeated addition. For example:

$$3(-2) = (-2) + (-2) + (-2) = -6$$

This suggests that the product of a positive number and a negative number is negative. Using the definition of opposites you can see that:

$$-3(-2) = -(3)(-2) = -(-6) = 6$$

This suggests that the product of two negative numbers is positive. The general rules for the sign of a product are given below.

RULES FOR THE SIGN OF A PRODUCT OF NONZERO NUMBERS

- A product is negative if it has an *odd* number of negative factors.
- A product is positive if it has an *even* number of negative factors.

Student Help

STUDY TIP

In Example 1 note that:

$(-2)^4 = (-2)(-2)(-2)(-2) = 16$

is not the same as:

$-2^4 = -(2^4) = (-1)(2)(2)(2)(2) = -16$

EXAMPLE 1 Multiply Real Numbers

a. $-4(5) = -20$ — One negative factor, so product is negative.

b. $-2(5)(-3) = 30$ — Two negative factors, so product is positive.

c. $-10(-0.2)(-4) = -8$ — Three negative factors, so product is negative.

d. $(-2)^4 = 16$ — Four negative factors, so product is positive.

Checkpoint Multiply Real Numbers

Find the product.

1. $3(-5)$ **2.** $-2(4)(5)$ **3.** $-\frac{1}{3}(-3)(-2)$ **4.** $(-2)^3$

The rules for the sign of a product are a consequence of the following properties of multiplication.

Student Help

STUDY TIP
Notice the similarities in the properties of multiplication and the properties of addition (p. 79).

PROPERTIES OF MULTIPLICATION

CLOSURE PROPERTY The product of any two real numbers is a unique real number.

ab is a unique real number **Example:** $4 \cdot 2 = 8$

COMMUTATIVE PROPERTY The order in which two numbers are multiplied does not change the product.

$ab = ba$ **Example:** $3(-2) = (-2)3$

ASSOCIATIVE PROPERTY The way you group three numbers when multiplying does not change the product.

$(ab)c = a(bc)$ **Example:** $(-6 \cdot 2)3 = -6(2 \cdot 3)$

IDENTITY PROPERTY The product of a number and 1 is the number.

$1 \cdot a = a$ **Example:** $1 \cdot (-4) = -4$

PROPERTY OF ZERO The product of a number and 0 is 0.

$0 \cdot a = 0$ **Example:** $0 \cdot (-2) = 0$

PROPERTY OF NEGATIVE ONE The product of a number and -1 is the opposite of the number.

$-1 \cdot a = -a$ **Example:** $-1 \cdot (-3) = 3$

Student Help

WRITING ALGEBRA
Example 2 shows an efficient method based on the Multiplication Property of Negative One. Fully written out Example 2(a) is as follows:

$$\begin{aligned} -2(-x) &= (-1)(2)(-1)(x) \\ &= (-1)(-1)(2x) \\ &= 2x \end{aligned}$$

EXAMPLE 2 Products with Variable Factors

Simplify the expression.

a. $-2(-x)$ **b.** $3(-n)(-n)(-n)$ **c.** $-1(-a)^2$

Solution

a. $-2(-x) = 2x$
Two minus signs, so product has no minus sign.

b. $3(-n)(-n)(-n) = 3(-n^3)$
Three minus signs, so product has a minus sign.

$= -3n^3$
One minus sign, so product has a minus sign.

c. $-1(-a)^2 = (-1)(-a)(-a)$
Write the power as a product.

$= (-1)(a^2)$
Two minus signs, so product has no minus sign.

$= -a^2$
Property of negative one

Checkpoint ✓ Products with Variable Factors

Simplify the expression.

5. $8(-t)$ **6.** $-x(-x)(-x)(-x)$ **7.** $-7(-b)^3$

Student Help

MORE EXAMPLES

More examples are available at www.mcdougallittell.com

EXAMPLE 3 Evaluate a Variable Expression

Evaluate $-4(-1)(-x)$ when $x = -5$.

Solution You can simplify the expression first, or substitute for x first.

$-4(-1)(-x) = -4x$ — Simplify expression first.

$= -4(-5)$ — Substitute -5 for x.

$= 20$ — Two negative factors, so product is positive.

$-4(-1)(-x) = -4(-1)[-(-5)]$ — Substitute -5 for x first.

$= -4(-1)(5)$ — Use definition of opposites.

$= 20$ — Two negative factors, so product is positive.

Checkpoint Evaluate a Variable Expression

Evaluate the expression when $x = -2$.

8. $-9(x)(-2)$ **9.** $3(4)(-x)$ **10.** $3(-x)^3$ **11.** $7(x^2)(-5)$

Link to Science

FLYING SQUIRRELS glide through the air using "gliding membranes," which are flaps of skin that extend from their wrists to their ankles.

EXAMPLE 4 Use Products in Real Life

FLYING SQUIRRELS A flying squirrel drops from a tree with a velocity of -6 feet per second. Find the *displacement*, which is the change in position, of the squirrel after 3.5 seconds.

Solution

VERBAL MODEL

Displacement = Velocity • Time

LABELS

Displacement = d (feet)

Velocity = -6 (feet per second)

Time = 3.5 (seconds)

ALGEBRAIC MODEL

$d = -6 \cdot 3.5$

$d = -21$

ANSWER The squirrel's change in position is -21 feet. The negative sign indicates downward motion.

Checkpoint Use Products in Real Life

12. A helicopter is descending at a velocity of -15 feet per second. Find the displacement of the helicopter after 4.5 seconds.

2.5 Exercises

Guided Practice

Vocabulary Check

Match the property with the statement that illustrates it.

1. Commutative property	**A.** $-1 \cdot 9 = -9$
2. Associative property	**B.** $4(-2) = (-2)4$
3. Identity property	**C.** $0 \cdot 8 = 0$
4. Property of zero	**D.** $1 \cdot (-15) = -15$
5. Property of negative one	**E.** $-7(5 \cdot 2) = (-7 \cdot 5)2$

Skill Check

Find the product.

6. $9(-1)$ **7.** $-5(7)$ **8.** $-4(-6)$ **9.** $(-1)^5$

Simplify the expression.

10. $-3(-6)(a)$ **11.** $5(-t)(-t)(-t)(-t)$ **12.** $6(-x)^3$

Evaluate the expression for the given value of the variable.

13. $2(-5)(-x)$ when $x = 4$ **14.** $6(-2)(x)$ when $x = -3$

Practice and Applications

CLOSURE PROPERTY **Tell whether the set is closed under the operation by deciding if the combination of any two numbers in the set of numbers is itself in the set.**

15. even integers under multiplication **16.** odd integers under addition

MULTIPLYING REAL NUMBERS **Find the product.**

17. $-7(4)$ **18.** $5(-5)$ **19.** $-6.3(2)$

20. $-7(-1.2)$ **21.** $-\frac{1}{2}\left(\frac{8}{3}\right)$ **22.** $-12\left(-\frac{1}{4}\right)$

23. $(-6)^3$ **24.** $(-4)^4$ **25.** $-(7)^2$

26. $-2(-5)(7)$ **27.** $6(9)(-1)$ **28.** $-5(-4)(-8)$

29. $2.7(-6)(-6)$ **30.** $-3.3(-1)(-1.5)$ **31.** $15\left(-\frac{2}{15}\right)\left(\frac{3}{4}\right)$

PRODUCTS WITH VARIABLE FACTORS **Simplify the expression.**

32. $-3(-y)$ **33.** $7(-x)$ **34.** $-2(k)$

35. $5(-a)(-a)(-a)$ **36.** $-8(z)(z)$ **37.** $-2(5)(-r)(-r)$

38. $(-b)^3$ **39.** $-2(-x)^2$ **40.** $-(-y)^4$

Student Help

HOMEWORK HELP
Example 1: Exs. 15–31
Example 2: Exs. 32–40
Example 3: Exs. 41–49
Example 4: Exs. 50–55

EVALUATING EXPRESSIONS **Evaluate the expression for the given value of the variable.**

41. $-8(d)$ when $d = 6$

42. $3(-4)(n)$ when $n = -2$

43. $-3(-a)(-a)$ when $a = -7$

44. $9(-2)(-r)^3$ when $r = 2$

45. $-4.1(-5)(h)$ when $h = 2$

46. $-2\left(\frac{11}{2}\right)(t)$ when $t = -3$

Student Help

LOOK BACK
For help with counterexamples, see p. 73

COUNTEREXAMPLES **Determine whether the statement is *true* or *false*. If it is false, give a counterexample.**

47. $(-a) \cdot (-b) = (-b) \cdot (-a)$

48. The product $(-a) \cdot (-1)$ is always positive.

49. If $a > b$, then $a \cdot 0$ is greater than $b \cdot 0$.

Link to Careers

MOUNTAIN GUIDES plan climbing expeditions. The guides also instruct students on basic climbing techniques, such as rappelling.

INTERNET More about mountain guides is available at www.mcdougallittell.com

MOUNTAIN RAPPELLING **You rappel down the side of a mountain at a rate of 2 feet per second.**

50. Write an algebraic model for your displacement d (in feet) after t seconds.

51. What is your change in position after rappelling for 10 seconds?

52. If the mountain is 40 feet high, how much farther must you rappel before you reach the ground?

Science Link **Scientists estimate that a peregrine falcon can dive for its prey at a rate of about 300 feet per second.**

53. Write an algebraic model for the displacement d (in feet) of a peregrine falcon after t seconds.

54. What is a peregrine falcon's change in position after diving for 2 seconds?

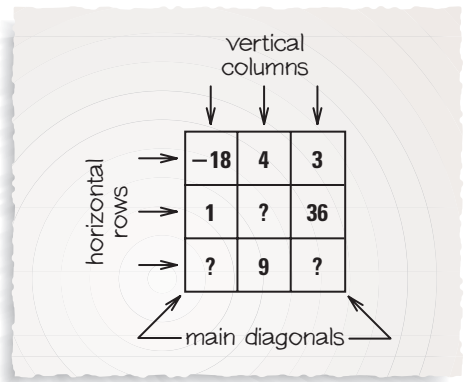
55. If the peregrine falcon spotted its prey 750 feet below, how much farther must it dive to reach its prey?

Puzzler **A multiplication magic square is a square in which the product of the numbers in every horizontal, vertical, and main diagonal line is constant.**

56. Find the constant of the magic square shown by multiplying the numbers in the first row of the square.

57. Copy and complete the magic square by finding the missing number in each column.

58. Check your answer by finding the product of each main diagonal.



Standardized Test Practice

59. MULTIPLE CHOICE What does $-3(6)\left(-\frac{1}{3}\right)$ equal?

Ⓐ -6 Ⓑ -2 Ⓒ 2 Ⓓ 6

60. MULTIPLE CHOICE Which of the following statements is *not* true?

Ⓕ The product of any number and zero is zero.

Ⓖ The order in which two numbers are multiplied does not matter.

Ⓗ The product of any number and -1 is a negative number.

Ⓙ The product of any number and -1 is the opposite of the number.

61. MULTIPLE CHOICE Simplify the expression $2(-4)(-x)(-x)(-x)$.

Ⓐ $-24x$ Ⓑ $-8x^3$ Ⓒ $8x^3$ Ⓓ $24x$

62. MULTIPLE CHOICE Evaluate $9(-x)^2(-2)$ when $x = 3$.

Ⓕ -162 Ⓖ -108 Ⓗ 108 Ⓙ 162

Mixed Review

MENTAL MATH **Use mental math to solve the equation.** *(Lesson 1.4)*

63. $6 + c = 8$ **64.** $x - 7 = 4$ **65.** $8 - a = 4$

66. $3z = 15$ **67.** $(m)(2) = 24$ **68.** $r \div 6 = 2$

LINE GRAPHS **Draw a line graph to represent the function given by the input-output table.** *(Lesson 1.8)*

69.

Input x	1	2	3	4	5	6
Output y	22	20	18	16	14	12

70.

Input x	1	2	3	4	5	6
Output y	0	5	10	15	20	25

FINDING ABSOLUTE VALUE **Evaluate the expression.** *(Lesson 2.2)*

71. $|2|$ **72.** $|-6|$ **73.** $-|9|$ **74.** $-|-7|$

75. $|-7.2|$ **76.** $-|6.8|$ **77.** $|10.43|$ **78.** $-|-0.05|$

FINDING TERMS **Identify the terms of the expression.** *(Lesson 2.4)*

79. $12 - z$ **80.** $-t + 5$ **81.** $4w - 11$

82. $31 - 15n$ **83.** $-7x + 4x$ **84.** $-3c - 4$

Maintaining Skills

LEAST COMMON MULTIPLE **Find the least common multiple of the numbers.** *(Skills Review p. 761)*

85. 4 and 5 **86.** 24 and 36 **87.** 30 and 25

88. 111 and 55 **89.** 312 and 210 **90.** 176 and 264