2.2 Absolute Value

Goal

Find the opposite and the absolute value of a number.

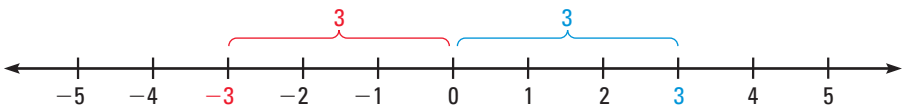
Key Words

- opposite
- absolute value
- counterexample

What is a launch pad elevator's velocity and speed?

Velocity and speed are different concepts. Velocity tells you how fast an object is moving and in what direction. It can be positive or negative. Speed tells you only how fast an object is moving. It can only be positive. In Example 4 you will find the velocity and speed of a launch pad elevator for a space shuttle.

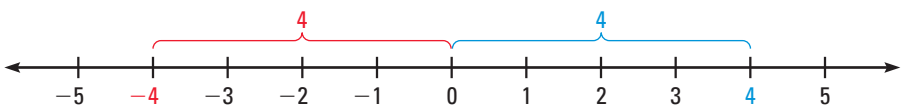
Two numbers that are the same distance from 0 on a number line but on opposite sides of 0 are **opposites**. The numbers -3 and 3 are opposites because each is 3 units from 0.



EXAMPLE 1 Find the Opposite of a Number

Use a number line to find the opposite of -4.

Solution



You can see that -4 is 4 units to the *left* of 0. The opposite of -4 is 4 units to the *right* of 0. So the opposite of -4 is 4.

ABSOLUTE VALUE The **absolute value** of a number is its distance from zero on a number line. The symbol $|a|$ represents the absolute value of a.

Student Help

READING ALGEBRA The expression $-a$ can be read as "negative a" or as "the opposite of a."

THE ABSOLUTE VALUE OF A NUMBER

- If a is a positive number, then $|a| = a$. **Example:** $|3| = 3$
- If a is zero, then $|a| = 0$. **Example:** $|0| = 0$
- If a is a negative number, then $|a| = -a$. **Example:** $|-3| = -(-3) = 3$

EXAMPLE 2 Find Absolute Value

Evaluate the expression.

a. $|5|$ **b.** $|-2.3|$ **c.** $-\left|\frac{1}{2}\right|$ **d.** $-|-8|$

Solution

a. $|5| = 5$ — If a is positive, then $|a| = a$.

b. $|-2.3| = -(-2.3)$ — If a is negative, then $|a| = -a$.
$= 2.3$ — Use definition of opposites.

c. $-\left|\frac{1}{2}\right| = -\left(\frac{1}{2}\right)$ — The absolute value of $\frac{1}{2}$ is $\frac{1}{2}$.
$= -\frac{1}{2}$ — Use definition of opposites.

d. $-|-8| = -(8)$ — The absolute value of -8 is 8.
$= -8$ — Use definition of opposites.

Checkpoint ✓ Find Absolute Value

Evaluate the expression.

1. $|-4|$ **2.** $|0|$ **3.** $\left|\frac{3}{2}\right|$ **4.** $-|1.7|$

Student Help

LOOK BACK
For help with the solution of an equation, see p. 24.

EXAMPLE 3 Solve an Absolute Value Equation

Use mental math to solve the equation.

a. $|x| = 7$ **b.** $|x| = 5.1$ **c.** $|x| = -\frac{2}{9}$

Solution

a. Ask, "What numbers are 7 units from 0?" Both 7 and -7 are 7 units from 0, so there are two solutions: 7 and -7.

b. Ask, "What numbers are 5.1 units from 0?" Both 5.1 and -5.1 are 5.1 units from 0, so there are two solutions: 5.1 and -5.1.

c. The absolute value of a number is never negative, so there is no solution.

Checkpoint ✓ Solve an Absolute Value Equation

Use mental math to solve the equation. If there is no solution, write *no solution*.

5. $|x| = -4$ **6.** $|x| = 1.5$ **7.** $|x| = \frac{1}{6}$

VELOCITY AND SPEED *Velocity* indicates both speed and direction (up is positive and down is negative). The *speed* of an object is the absolute value of its velocity.

EXAMPLE 4 Find Velocity and Speed

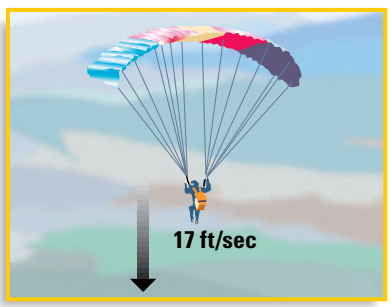
SCIENCE LINK A launch pad elevator for a space shuttle drops at a rate of about 12 feet per second. What are its velocity and speed?

Solution

Velocity $= -12$ feet per second — Motion is downward.

Speed $= |-12| = 12$ feet per second — Speed is never negative.

Checkpoint Find Velocity and Speed



A parachutist descends at a rate of about 17 feet per second.

8. What is the parachutist's velocity?

9. What is the parachutist's speed?

COUNTEREXAMPLE To prove that a statement is true, you need to show that it is true for *all* examples. To prove that a statement is false, it is enough to show that it is *not* true for a single example, called a **counterexample**.

Student Help

MORE EXAMPLES

More examples are available at www.mcdougallittell.com

EXAMPLE 5 Use a Counterexample

Determine whether the statement is *true* or *false*. If it is false, give a counterexample.

a. The opposite of a number is always negative.

b. The absolute value of a number is never negative.

Solution

a. False. Counterexample: The opposite of -5 is 5, which is positive.

b. True, by definition.

Checkpoint Use a Counterexample

Determine whether the statement is *true* or *false*. If it is false, give a counterexample.

10. The expression $-a$ is never positive.

11. The expression $|a|$ is always greater than or equal to a.

12. The absolute value of a negative number is always negative.

2.2 Exercises

Guided Practice

Vocabulary Check

1. What is the opposite of 2?

2. Complete: The absolute value of a number is its distance from __?__ on a number line.

Skill Check

Find the opposite of the number.

3. 1 **4.** -3 **5.** -2.4 **6.** $\frac{1}{2}$

Evaluate the expression.

7. $|-12|$ **8.** $|6|$ **9.** $-|5.1|$ **10.** $\left|-\frac{1}{5}\right|$

Use mental math to solve the equation. If there is no solution, write *no solution*.

11. $|x| = 8$ **12.** $|x| = -9$ **13.** $|x| = 5.5$ **14.** $|x| = \frac{2}{3}$

Determine whether the statement is *true* or *false*. If it is false, give a counterexample.

15. The opposite of a number is always less than the number.

16. The absolute value of a number is always positive or zero.

Practice and Applications

FINDING OPPOSITES **Find the opposite of the number.**

17. 8 **18.** -3 **19.** -10 **20.** 0

21. -3.8 **22.** 2.5 **23.** $-\frac{1}{9}$ **24.** $\frac{5}{6}$

FINDING ABSOLUTE VALUE **Evaluate the expression.**

25. $|7|$ **26.** $|-4|$ **27.** $-|3|$ **28.** $-|-2|$

29. $|-0.8|$ **30.** $|-4.5|$ **31.** $\left|\frac{2}{3}\right|$ **32.** $-\left|-\frac{8}{9}\right|$

Student Help

HOMEWORK HELP
Example 1: Exs. 17–24
Example 2: Exs. 25–32, 41, 42
Example 3: Exs. 33–40
Example 4: Exs. 43–50
Example 5: Exs. 51–53

SOLVING AN EQUATION **Use mental math to solve the equation. If there is no solution, write *no solution*.**

33. $|x| = 4$ **34.** $|x| = 0$ **35.** $|x| = -2$ **36.** $|x| = 1$

37. $|x| = 3.7$ **38.** $|x| = -9.6$ **39.** $|x| = \frac{11}{2}$ **40.** $|x| = \frac{5}{6}$

Link to Science

SATURN radiates more energy into space than it receives from the sun, resulting in a constant average surface temperature.

Science Link **In Exercises 41 and 42, use the table at the right which shows the average high and low surface temperatures for the planets in our solar system.**

41. The range of temperatures for Mercury and Mars is the sum of the absolute values of the high and low temperatures. Find the range of temperatures for these planets.

42. The range of temperatures for the other planets is the difference of the absolute values of the high and low temperatures. Find the range of temperatures for these planets.

Planet	High (°F)	Low (°F)
Mercury	800	−280
Venus	847	847
Earth	98	8
Mars	98	−190
Jupiter	−244	−244
Saturn	−301	−301
Uranus	−353	−353
Neptune	−373	−373
Pluto	−393	−393

VELOCITY **Determine whether to use a *positive* or a *negative* number to represent the velocity.**

43. The velocity of a descending hot-air balloon

44. The velocity of a rising rocket

45. The velocity of a kite as it lifts into the air

46. The velocity of a falling meteorite

VELOCITY AND SPEED **A helicopter is descending at a rate of 6 feet per second.**

47. What is the helicopter's velocity? **48.** What is the helicopter's speed?

VELOCITY AND SPEED **The elevator in the Washington Monument in Washington, D.C., climbs at a rate of about 400 feet per minute.**

49. What is the elevator's velocity? **50.** What is the elevator's speed?

Student Help

HOMEWORK HELP

INTERNET Extra help with problem solving in Exs. 51–53 is available at www.mcdougallittell.com

USING COUNTEREXAMPLES **Determine whether the statement is *true* or *false*. If it is false, give a counterexample.**

51. The opposite of $-a$ is always positive.

52. The opposite of $|a|$ is never positive.

53. The expression $|-a|$ is never negative.

CHALLENGE **Determine whether the statement is *always, sometimes,* or *never* true. Explain.**

54. The absolute value of a number is the same as the absolute value of the opposite number. In other words, $|x| = |-x|$.

55. The opposite of the absolute value of a number is the same as the absolute value of the opposite of the number. In other words, $-|x| = |-x|$.

Standardized Test Practice

56. MULTIPLE CHOICE What is the opposite of 5?

Ⓐ $\frac{1}{5}$ Ⓑ $-\frac{1}{5}$ Ⓒ 5 Ⓓ -5

57. MULTIPLE CHOICE What is the value of $-|-2|$?

Ⓕ 2 Ⓖ -2 Ⓗ $|2|$ Ⓙ $|-2|$

58. MULTIPLE CHOICE What is the solution of $|x| = 18$?

Ⓐ 18 Ⓑ -18

Ⓒ 18 and -18 Ⓓ none of these

59. MULTIPLE CHOICE What is the velocity of a diver who descends to the ocean floor at a rate of 3 meters per second?

Ⓕ $|-3|$ m/sec Ⓖ $|3|$ m/sec

Ⓗ -3 m/sec Ⓙ 3 m/sec

Mixed Review

EVALUATING EXPRESSIONS **Evaluate the expression for the given value of the variable.** *(Lesson 1.1)*

60. $x + 3$ when $x = 2$ **61.** $a - 7$ when $a = 10$

62. $3y$ when $y = 0$ **63.** $(t)(5)$ when $t = 15$

64. $\frac{z}{2}$ when $z = 8$ **65.** $\frac{9}{p}$ when $p = 3$

TRANSLATING SENTENCES **Write the sentence as an equation or an inequality. Let *x* represent the number.** *(Lesson 1.5)*

66. 5 less than a number is 8.

67. 8 more than a number is 17.

68. The quotient of 15 and a number is greater than or equal to 3.

69. 9 times a number is less than 6.

COMPARING NUMBERS **Graph the numbers on a number line. Then write two inequalities that compare the numbers.** *(Lesson 2.1)*

70. $7, -7$ **71.** $-2, -6$ **72.** $-10, -1$

73. $0.4, -3$ **74.** $2.2, -3.3$ **75.** $-10, -\frac{1}{10}$

Maintaining Skills

SUBTRACTING FRACTIONS **Subtract. Write the answer in simplest form.** *(Skills Review p. 764)*

76. $\frac{3}{4} - \frac{1}{4}$ **77.** $\frac{7}{9} - \frac{2}{9}$ **78.** $\frac{7}{10} - \frac{3}{10}$

79. $\frac{14}{15} - \frac{4}{15}$ **80.** $\frac{25}{27} - \frac{16}{27}$ **81.** $\frac{41}{44} - \frac{19}{44}$