1.8 An Introduction to Functions

Goal

Use four different ways to represent functions.

Key Words

- function
- input
- output
- input-output table
- domain
- range

What is the altitude of the balloon?

You are in a hot-air balloon. You rise at a steady rate of 20 feet per minute. In Example 2 you will use the relationship between time and height to find the altitude of the balloon after a given number of minutes.

A **function** is a rule that establishes a relationship between two quantities, called the **input** and the **output**. For each input, there is exactly one output—even though two different inputs may give the same output.

One way to describe a function is to make an **input-output table**.

EXAMPLE 1 Make an Input-Output Table

GEOMETRY LINK The diagram shows the first six triangular numbers.

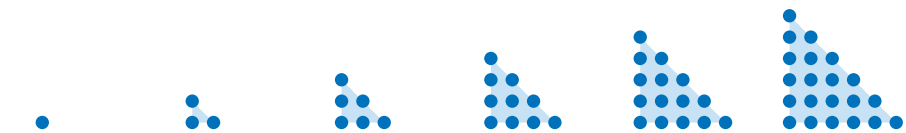


Figure 1 | Figure 2 | Figure 3 | Figure 4 | Figure 5 | Figure 6

a. Using the first six figures, make an input-output table in which the input is the figure number n and the output is the triangular number T.

b. Does the table represent a function? Justify your answer.

Solution

a. Use the diagram to make an input-output table, as shown below.

Input n	1	2	3	4	5	6
Output T	1	3	6	10	15	21

b. This is a function, because for each input there is exactly one output.

DOMAIN AND RANGE The collection of all input values is the **domain** of the function and the collection of all output values is the **range** of the function. The domain of the function in Example 1 is 1, 2, 3, 4, 5, 6; the range of the function is 1, 3, 6, 10, 15, 21.

When you are given the rule for a function, you can prepare to graph the function by making a table showing numbers in the domain and their corresponding output values.

EXAMPLE 2 Use a Table to Graph a Function

BALLOONING You are at an altitude of 250 feet in a hot-air balloon. You turn on the burner and rise at a rate of 20 feet per minute for 5 minutes. Your altitude h in feet after you have risen for t minutes is given by

$$h = 250 + 20t, \text{ where } t \geq 0 \text{ and } t \leq 5.$$

a. Use the function to find the output h in feet for several inputs. Then organize the data into an input-output table.

b. Use the data in the table to draw a graph that represents the function.

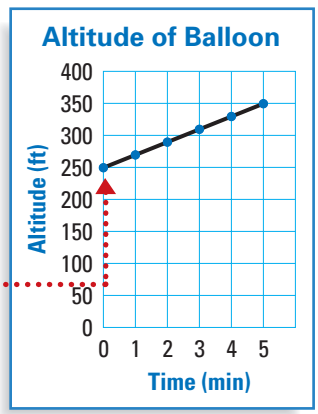
Solution

a. Find the outputs for $t = 0, 1, 2, 3, 4,$ and 5. Then make a table.

INPUT (MINUTES)	FUNCTION	OUTPUT (FEET)
$t = 0$	$h = 250 + 20(0)$	$h = 250$
$t = 1$	$h = 250 + 20(1)$	$h = 270$
$t = 2$	$h = 250 + 20(2)$	$h = 290$
$t = 3$	$h = 250 + 20(3)$	$h = 310$
$t = 4$	$h = 250 + 20(4)$	$h = 330$
$t = 5$	$h = 250 + 20(5)$	$h = 350$

Input t	0	1	2	3	4	5
Output h	250	270	290	310	330	350

b.



Let the horizontal axis represent the input t (in minutes). Label the axis from 0 to 5. Let the vertical axis represent the output h (in feet). Label the axis from 0 to 400.

Plot the data points given in the table. Finally, connect the points.

The graph shows that as the time increases, the height of the balloon increases.

The graph represents the function $h = 250 + 20t$, where $t \geq 0$ and $t \leq 5$.

Student Help

STUDY TIP
To plot the first point (t, h) find $t = 0$ on the horizontal axis. Then find $h = 250$ on the vertical axis. Mark the point (0, 250).

Student Help

MORE EXAMPLES

More examples are available at www.mcdougallittell.com

EXAMPLE 3 Write an Equation to Represent a Function

SCUBA DIVING As you dive deeper and deeper into the ocean, the pressure of the water on your body steadily increases. The pressure at the surface of the water is 14.7 pounds per square inch (psi). The pressure increases at a rate of 0.445 psi for each foot you descend. Write an equation to represent the pressure P as a function of the depth d for every 20 feet you descend until you reach a depth of 60 feet.

Solution

VERBAL MODEL

Pressure at given depth = **Pressure at surface** + **Rate of change in pressure** • **Diving depth**

LABELS

Pressure at given depth = P (psi)

Pressure at surface = **14.7** (psi)

Rate of change in pressure = **0.445** (psi per foot of depth)

Diving depth = d (feet)

ALGEBRAIC MODEL

$P = 14.7 + 0.445d$ where $d \geq 0$ and $d \leq 60$

ANSWER The function can be represented by the equation $P = 14.7 + 0.445d$, where $d \geq 0$ and $d \leq 60$.

Checkpoint ✓ Represent a Function

Use the algebraic model from Example 3.

1. Make an input-output table for the function. Use $d = 0$, 20, 40, and 60.

2. Draw a graph that represents the function.

SUMMARY

Four Ways to Represent Functions

INPUT-OUTPUT TABLE

Input n	Output P
1	1
2	3
3	6
4	10
5	15
6	21

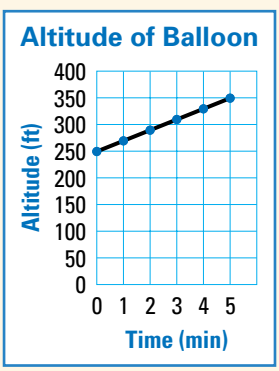
WORDS

You are in a hot-air balloon at a height of 250 feet. You begin to rise higher at a rate of 20 feet per minute for a period of 5 minutes.

EQUATION

$h = 250 + 20t$, where $t \geq 0$ and $t \leq 5$

GRAPH



1.8 Exercises

Guided Practice

Vocabulary Check

Complete the sentence.

1. A function is a relationship between two quantities, called the __?__ and the __?__.

2. The collection of all input values is the __?__ of the function.

3. The collection of all output values is the __?__ of the function.

Skill Check

CAMPING In Exercises 4–6, use the following information.
You are going camping. The cost for renting a cabin at Shady Knoll Campground is $65.00 plus $12.00 per person. The cost in dollars is

$$C = 65 + 12n, \text{ where } n \text{ is the number of people.}$$

4. Copy and complete the input-output table.

Input n	1	2	3	4	5	6
Output C	?	?	?	?	?	?

5. Draw a graph that is made up of isolated points representing the cost of renting a cabin.

6. Determine the range of the function from the given input values in the input-output table.

Practice and Applications

INPUT-OUTPUT TABLES Make an input-output table for the function. Use 0, 1, 2, 3, 4, and 5 as values for x.

7. $y = 6x + 5$ **8.** $y = 26 - 2x$ **9.** $y = (x + 3) \cdot 7$

10. $y = 85 - 15x$ **11.** $y = 5(15 - x)$ **12.** $y = 2(6x + 10)$

LINE GRAPHS Draw a line graph to represent the function given by the input-output table.

13.

Input x	1	2	3	4	5	6
Output y	14	12	10	8	6	4

14.

Input x	1	2	3	4	5	6
Output y	8	11	14	17	20	23

Student Help

HOMEWORK HELP
Example 1: Exs. 7–12, 15–20, 23, 24
Example 2: Exs. 13–15, 21, 23–25
Example 3: Exs. 23, 24

15. **Science Link** The distance d (in miles) that sound travels in air in time t (in seconds) is represented by the function $d = 0.2t$. Make a table of the input t and the output d. Use t values of 0, 5, 10, 15, 20, 25, and 30. Use your table to help you draw the graph of the function.

Student Help

HOMEWORK HELP

INTERNET Extra help with problem solving in Exs. 16–19 is available at www.mcdougallittell.com

CRITICAL THINKING **Determine whether the table represents a function.**

16.

Input	Output
1	3
2	4
3	5

17.

Input	Output
1	2
3	3
3	4

18.

Input	Output
2	2
3	4
4	6

19.

Input	Output
1	3
1	4
2	5

CAR RACING **In Exercises 20–22, use the following information.**
The fastest winning speed in the Daytona 500 is about 178 miles per hour. In the table below, calculate the distance traveled d (in miles) after time t (in hours) using the equation $d = 178t$.

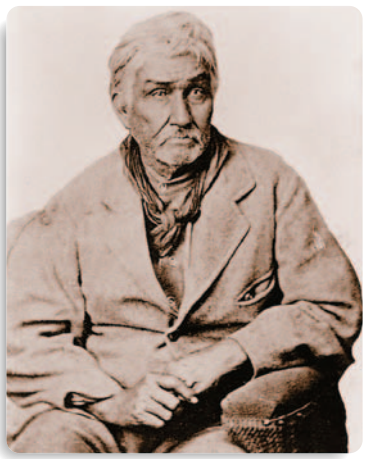
20. Copy and complete the input-output table.

Time (hours)	0.25	0.50	0.75	1.00	1.25	1.50
Distance traveled (miles)	?	?	?	?	?	?

21. Use the data to draw a graph.

22. For what values of t does the formula $d = 178t$ correspond to the situation being modeled?

Link to History



JESSE CHISHOLM, the person for whom the Chisholm Trail is named, was a trader who was part Cherokee.

23. **History Link** In 1866 Texas cowhands used the Chisholm Trail to drive cattle north to the railroads in Kansas. The average rate r that the cattle could be moved along the trail was 11 miles per day.

a. Write an equation, where d is distance and t is time in days.

b. Make a table of input t and output d for $t = 7$, 14, and 28. Then graph the data.

c. The distance d from San Antonio to Abilene was about 1100 miles. How long did it take to drive cattle the entire length of the trail?

Link to Careers

SCUBA DIVERS must take an instructional class in order to become certified.

24. SCUBA DIVERS While you are on vacation, you want to rent scuba equipment. It costs about \$90 a day to rent the equipment. Find the cost of renting equipment for 1, 2, 3, and 4 days.

a. Write an equation where R is the total rental cost and n is the number of days. Make an input-output table.

b. Draw a graph that represents the function.

25. WATER TEMPERATURE The table below gives the temperature of water as it cools. Using this table, draw a graph that estimates the temperature of the water for $t \geq 0$ and $t \leq 25$.

Time (minutes)	0	5	10	15	20	25
Temperature (°C)	100	90	81	73	66	60

26. CHALLENGE The function $y = x^2$ has a U-shaped graph called a *parabola*. If the domain of this function is given as $x \geq 0$ and $x \leq 4$, find the range.

Standardized Test Practice

27. MULTIPLE CHOICE Which table does *not* represent a function?

Ⓐ

Input	Output
1	3
2	3
3	3
4	3

Ⓑ

Input	Output
1	2
2	4
3	6
4	8

Ⓒ

Input	Output
5	4
6	4
7	5
8	5

Ⓓ

Input	Output
5	1
5	3
6	1
6	3

28. MULTIPLE CHOICE Which function has an output of $j = 27$ for an input of $a = 3$?

Ⓕ $j = 4a + 15$ Ⓖ $j = 15a + 4$ Ⓗ $j = 15 \cdot 4a$ Ⓙ $j = 27a$

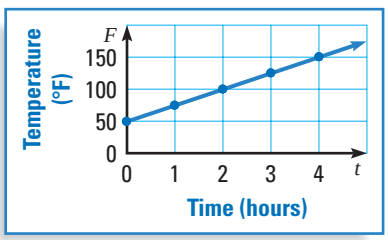
29. MULTIPLE CHOICE Which function is best represented by the graph?

Ⓐ $F = 50 + 25t$

Ⓑ $F = 25 + t$

Ⓒ $F = 25 + 50t$

Ⓓ $F = 25t$



Mixed Review

EVALUATING EXPRESSIONS **Evaluate the variable expression when $a = 3$ and $c = 5$.** *(Lessons 1.1 and 1.2)*

30. $a + c$ **31.** $(a + c)^2$ **32.** $a^2 + c^2$

33. ac **34.** $a \cdot (c^2)$ **35.** $(a^2) \cdot c$

36. TRANSLATING PHRASES Write a variable expression for the phrase *9 decreased by a number n.* *(Lesson 1.5)*

37. TRANSLATING SENTENCES Write the inequality for the sentence: *The quotient of 72 and a number x is greater than 7.* *(Lesson 1.5)*

Maintaining Skills

ADDING FRACTIONS **Add. Write the answer as a fraction or a mixed number in simplest form.** *(Skills Review p. 764)*

38. $\frac{2}{9} + \frac{8}{9}$ **39.** $\frac{5}{12} + \frac{1}{12}$ **40.** $\frac{12}{15} + \frac{7}{15}$

41. $\frac{11}{3} + \frac{2}{3}$ **42.** $\frac{5}{6} + \frac{7}{6}$ **43.** $\frac{2}{8} + \frac{1}{8}$

44. $\frac{3}{5} + \frac{1}{5}$ **45.** $\frac{3}{4} + \frac{9}{4}$ **46.** $\frac{9}{14} + \frac{3}{14}$

Quiz 3

1. RECYCLING A recycling center pays 5¢ apiece for aluminum cans and certain glass bottles. Jean has four cans and the total amount paid for her collection of cans and bottles is 50¢. Use a verbal model to find how many glass bottles are in Jean's collection. *(Lesson 1.6)*

ARTS ACTIVITIES **In Exercises 2 and 3, use the table showing the percent of 18-to-24-year-olds that attended various arts activities at least once a year.** *(Lesson 1.7)*

Arts Activities Attended by 18-to-24-year-olds				
Jazz	Musical play	Non-musical play	Art museum	Historic park
15%	26%	20%	38%	46%

2. Make a bar graph of the data.

3. What conclusions can you draw from the bar graph?

HOT-AIR BALLOONS **You are at an altitude of 200 feet in a hot-air balloon. You rise at a rate of 25 feet per minute for 4 minutes. Your altitude h (in feet) after you have risen for t minutes is given by**

$$h = 200 + 25t, \text{ where } t \geq 0 \text{ and } t \leq 4.$$ *(Lesson 1.8)*

4. Make an input-output table using 0, 1, 2, 3, and 4 as values for x.

5. Use your table to draw a graph that represents the function.

6. Determine the range of the function.