1.4 Equations and Inequalities

How much do the ingredients cost?

Goal
Check solutions of equations and inequalities.

Key Words
- equation
- solution
- inequality

You can use an *equation* to solve a real-life problem. In Example 3 you will use an equation to estimate the cost of ingredients for nachos.

Student Help

VOCABULARY TIP
Equation comes from a Latin word that means "to be equal".

An **equation** is a statement formed by placing an equal sign (=) between two expressions. An equation has a left side and a right side.

Left side → $4x + 1 = 9$ ← Right side

$4x + 1 = 9$ is an Equation

When the variable in an equation is replaced by a number, the resulting statement is either true or false. If the statement is true, the number is a **solution** of the equation.

EXAMPLE 1 Check Possible Solutions

Check to see if 2 and 3 are solutions of the equation $4x + 1 = 9$.

Solution

Substitute the x values 2 and 3 into the equation. If both sides of the equation are equal in value, then the number is a solution.

X VALUE	SUBSTITUTE	SIMPLIFY	CONCLUSION
2	$4(2) + 1 \stackrel{?}{=} 9$	$9 = 9$	True, 2 is a solution.
3	$4(3) + 1 \stackrel{?}{=} 9$	$13 \neq 9$	False, 3 is *not* a solution.

$13 \neq 9$ is not equal to

ANSWER The number 2 is a solution of the equation $4x + 1 = 9$, because the statement is true. The number 3 is not a solution, because the statement is false.

SOLVING EQUATIONS Finding all the solutions of an equation is called *solving* the equation. Some equations are simple enough to be solved with mental math. Later in the book you will learn how to systematically solve more complex equations.

Student Help

MORE EXAMPLES

INTERNET More examples are available at www.mcdougallittell.com

EXAMPLE 2 Solve Equations with Mental Math

To solve equations with mental math, think of the equation as a question.

EQUATION	QUESTION	SOLUTION
$2x = 10$	2 times what number gives 10?	$2 \cdot 5 = 10$, so $x = 5$
$4 = x - 3$	4 is equal to what number minus 3?	$4 = 7 - 3$, so $x = 7$
$2 + x = 6$	2 plus what number gives 6?	$2 + 4 = 6$, so $x = 4$
$\frac{x}{3} = 1$	What number divided by 3 gives 1?	$\frac{3}{3} = 1$, so $x = 3$

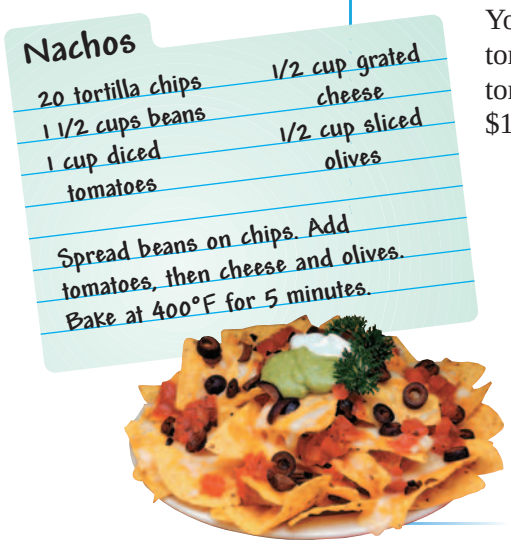
Then check each solution by substituting the number in the original equation. If the statement is true, the number is a solution.

Checkpoint Solve Equations and Check Solutions

Use mental math to solve the equation. Then check your solution.

1. $2 = 6 - x$ **2.** $x + 3 = 11$ **3.** $\frac{x}{4} = 5$ **4.** $14 = 2x$

EXAMPLE 3 Use Mental Math to Solve a Real-Life Equation



You are buying ingredients for nachos. At the market you find that tortilla chips cost \$2.99, beans cost \$.99, cheese costs \$3.99, two tomatoes cost \$1.00, and olives cost \$1.49. There is no tax. You have \$10. About how much more money do you need?

Solution

Ask: The total cost equals 10 plus what number of dollars? Let x represent the extra money you need. Use rounding to estimate the total cost.

$$3 + 1 + 4 + 1 + 1.5 = 10 + x$$

$$10.5 = 10 + x$$

ANSWER The total cost is about 10.5 or \$10.50, so you need about \$.50 more to purchase all the ingredients.

Checkpoint Use Mental Math to Solve a Real-Life Equation

5. Solve the equation in Example 3 if a large bag of chips costs \$3.99. About how much more money would you need to buy the nacho ingredients?

Student Help

STUDY TIP
The "wide end" of the inequality symbol faces the greater number. For help with comparing numbers, see p. 770.

An **inequality** is a statement formed by placing an inequality symbol, such as $<$, between two expressions.

INEQUALITY SYMBOL	MEANING	EXAMPLE
$<$	is less than	$1 + 3 < 5$
$\leq$	is less than or equal to	$6 - 1 \leq 5$
$>$	is greater than	$10 > 2(4)$
$\geq$	is greater than or equal to	$10 \geq 9 - 1$

For inequalities involving a single variable, a solution is a number that produces a true statement when it is substituted for the variable in the inequality.

EXAMPLE 4 Check Solutions of Inequalities

Check to see if $x = 4$ is or is not a solution of the inequality.

INEQUALITY	SUBSTITUTE	SIMPLIFY	CONCLUSION
$x + 3 \geq 9$	$4 + 3 \stackrel{?}{\geq} 9$	$7 \not\geq 9$	False, 4 is *not* a solution.
$2x - 1 < 8$	$2(4) - 1 \stackrel{?}{<} 8$	$7 < 8$	True, 4 is a solution.

Checkpoint Check Solutions of Inequalities

Check to see if the value of n is or is not a solution of $3n - 4 \leq 8$.

6. $n = 2$ **7.** $n = 3$ **8.** $n = 4$ **9.** $n = 5$

Link to Careers

VETERINARIANS specialize in the health care of either small animals, such as cats, or large animals, such as horses.

EXAMPLE 5 Check Solutions in Real Life

VETERINARIANS Your vet tells you to restrict your cat's caloric intake to less than or equal to 500 calories a day. Two times a day, you give your cat a serving of food that has x calories. Does 250 calories for each serving meet the vet's restriction?

Solution

1. ***Write*** the inequality. $2x \leq 500$
2. ***Substitute*** 250 for x. $2(250) \stackrel{?}{\leq} 500$
3. ***Simplify*** by multiplying. $500 \leq 500$

ANSWER Yes, 250 calories per serving meets the vet's restriction.

Checkpoint Check Solutions in Real Life

10. Check to see if 300 calories per serving meets the vet's restriction in Example 5.

1.4 Exercises

Guided Practice

Vocabulary Check

Explain if the following is an expression, an equation, or an inequality.

1. $3x + 1 = 14$ **2.** $7y - 6$ **3.** $5(y^2 + 4) - 7$

4. $5x - 1 = 3 + x$ **5.** $3x + 2 \leq 8$ **6.** $5x > 20$

7. Complete: An x value of 4 is a __?__ of the equation $x + 1 = 5$, because $4 + 1 = 5$.

Skill Check

Check to see if $a = 5$ is or is not a solution of the equation.

8. $a + 8 = 13$ **9.** $27 = 36 - 2a$ **10.** $a - 0 = 5$

11. $2a + 1 = 11$ **12.** $6a - 5 = 15$ **13.** $5a + 4 = 26$

14. $45 \div a = 9$ **15.** $a^2 + 2 = 27$ **16.** $\frac{40}{a} = 8$

Check to see if $b = 8$ is or is not a solution of the inequality.

17. $b + 10 > 19$ **18.** $14 - b \leq 3$ **19.** $5b > 35$

20. $8 \geq 64 \div b$ **21.** $3b - 24 > 0$ **22.** $16 \leq b^2$

23. $60 > 7b + 3$ **24.** $18 - b < 10$ **25.** $37 \geq 4b$

Practice and Applications

CHECKING SOLUTIONS OF EQUATIONS **Check to see if the given value of the variable is or is not a solution of the equation.**

26. $3b + 1 = 13$; $b = 4$ **27.** $5r - 10 = 11$; $r = 5$

28. $4c + 2 = 10$; $c = 2$ **29.** $6d - 5 = 31$; $d = 6$

30. $5 + x^2 = 17$; $x = 3$ **31.** $2y^3 + 3 = 5$; $y = 1$

32. $9 + 2t = 15$; $t = 12$ **33.** $n^2 - 5 = 20$; $n = 5$

SOLVING WITH MENTAL MATH **Use mental math to solve the equation.**

34. $x + 3 = 8$ **35.** $n + 6 = 11$ **36.** $p - 13 = 20$

37. $r - 1 = 7$ **38.** $3y = 12$ **39.** $4p = 36$

40. $z \div 4 = 5$ **41.** $\frac{x}{7} = 3$ **42.** $2b = 28$

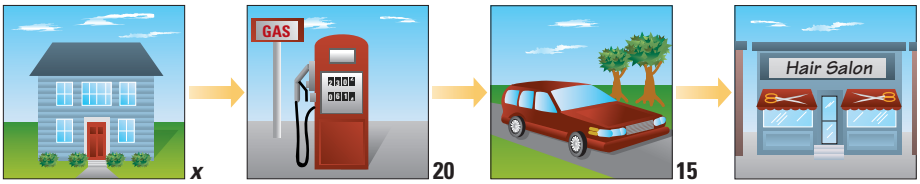
43. $11t = 22$ **44.** $29 - d = 10$ **45.** $3 + y = 8$

46. $r + 30 = 70$ **47.** $\frac{42}{x} = 7$ **48.** $7m = 49$

Student Help

HOMEWORK HELP
Example 1: Exs. 26–33
Example 2: Exs. 34–48
Example 3: Exs. 49, 50
Example 4: Exs. 51–56
Example 5: Exs. 57, 58

49. TIME MANAGEMENT You have a hair appointment in 60 minutes. It takes 20 minutes to get to the gas station and fill your tank. It takes 15 minutes to go from the gas station to the hair stylist. You wait x minutes before leaving your house and arrive on time for your appointment. Use the diagram to help decide which equation best models the situation.



A. $20 + 15 - x = 60$ **B.** $60 + 20 + 15 = x$

C. $60 - 20 + 15 + x = 60$ **D.** $x + 20 + 15 = 60$

50. MENTAL MATH Solve the equation you chose in Exercise 49.

CHECKING SOLUTIONS OF INEQUALITIES **Check to see if the given value of the variable is or is not a solution of the inequality.**

51. $n - 2 < 6$; $n = 3$ **52.** $a - 7 \geq 15$; $a = 22$

53. $6 + y \leq 8$; $y = 3$ **54.** $s + 5 > 8$; $s = 4$

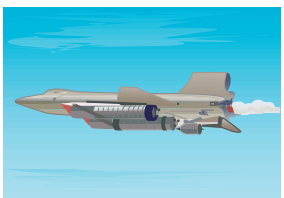
55. $7g \geq 47$; $g = 7$ **56.** $72 \div t > 6$; $t = 12$

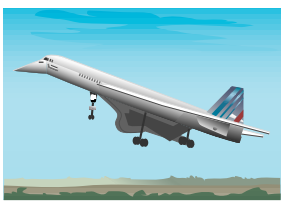
57. SELLING CARDS Your community center is selling cards. Your goal is to sell \$100 worth of cards. Each box sells for \$3. Using mental math, solve the inequality $3b \geq 100$ to determine at least how many boxes you must sell to meet your goal.

58. BUYING A GUITAR You are budgeting money to buy a guitar that costs \$150 including tax. If you save \$20 per month, will you have enough money in 6 months? Use the inequality $20n \geq 150$ to model the situation, where n represents the number of months.

59. Science Link *Mach number* is the maximum speed at which a plane can fly divided by the speed of sound. Copy and complete the table. Use the equation $m = \frac{v}{660}$, where m is the Mach number and v is the speed (in miles per hour) of the aircraft, to find the Mach number for each type of aircraft.

Airplane type	Test aircraft	Supersonic	Jet
Speed v	4620	1320	660
Mach number m	?	?	?


Test aircraft


Supersonic aircraft


Jet aircraft

Link to Science

CHUCK YEAGER in 1947 became the first person to fly faster than the speed of sound (Mach 1) or about 660 miles per hour.

60. **Puzzler** Use mental math to fill in the missing number so that all the equations have the number 6 as a solution.

a. $\boxed{?} + x = 18$ **b.** $\boxed{?}\, x = 30$ **c.** $\frac{\boxed{?}}{x} = 6$

Standardized Test Practice

61. **MULTIPLE CHOICE** Which is a solution of the equation $5(8 - x) = 25$?

(A) 2 (B) 3 (C) 4 (D) 5

62. **MULTIPLE CHOICE** For which inequality is $x = 238$ a solution?

(F) $250 \geq x + 12$ (G) $250 < x + 12$

(H) $250 > x + 12$ (J) $250 \leq x + 1$

63. **MULTIPLE CHOICE** The width of a soccer field cannot be greater than 100 yards. The area cannot be greater than 13,000 square yards. Which of the following would you use to find the possible length x of a soccer field?

(A) $100x \geq 13{,}000$ (B) $100x \leq 13{,}000$

(C) $100 + x \leq 13{,}000$ (D) $100x = 13{,}000$

Mixed Review

EVALUATING EXPRESSIONS **Evaluate the expression for the given value of the variable.** *(Lesson 1.1)*

64. $b - 12$ when $b = 43$ **65.** $12 + x$ when $x = 4$

66. $12n$ when $n = 4$ **67.** $\frac{y}{15}$ when $y = 30$

WRITING POWERS **Write the expression in exponential form.** *(Lesson 1.2)*

68. $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$ **69.** seven squared **70.** $y \cdot y \cdot y \cdot y$

71. $9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9$ **72.** twelve cubed **73.** $8d \cdot 8d \cdot 8d$

NUMERICAL EXPRESSIONS **Evaluate the expression. Then simplify the answer.** *(Lesson 1.3)*

74. $9 + 12 - 4$ **75.** $7 + 56 \div 8 - 2$ **76.** $63 \div 3 \cdot 3$

77. $4 \cdot 2 - 5$ **78.** $3 + 13 - 6$ **79.** $49 \div 7 + 2$

80. $(28 \div 4) + 3^2$ **81.** $\frac{4^2 + 2}{2}$ **82.** $2[(2 + 3)^2 - 10]$

Maintaining Skills

ROUNDING **Round the number to the underlined place value.** *(Skills Review p. 774)*

83. 5.64 **84.** 0.2625 **85.** 0.45695

86. 15.295 **87.** 758.949 **88.** 32.6582

89. 0.325 **90.** 26.96 **91.** 4.0965