1.3 Order of Operations

Goal
Use the established order of operations.

Key Words
- order of operations
- left-to-right rule

How many points ahead are you?

You are playing basketball. You make 8 field goals and 2 free throws. Your friend makes half as many field goals as you and no free throws. You will find how many points ahead you are in Example 5.

ORDER OF OPERATIONS In arithmetic and algebra there is an **order of operations** to evaluate an expression involving more than one operation.

ORDER OF OPERATIONS

STEP ① First do operations that occur within grouping symbols.

STEP ② Then evaluate powers.

STEP ③ Then do multiplications and divisions from left to right.

STEP ④ Finally, do additions and subtractions from left to right.

EXAMPLE 1 Evaluate Without Grouping Symbols

Evaluate the expression $3x^2 + 1$ when $x = 4$. Use the order of operations.

Solution

$$3x^2 + 1 = 3 \cdot 4^2 + 1 \qquad \text{Substitute 4 for } x.$$
$$= 3 \cdot 16 + 1 \qquad \text{Evaluate power.}$$
$$= 48 + 1 \qquad \text{Multiply 3 times 16.}$$
$$= 49 \qquad \text{Add.}$$

Checkpoint ✓ *Evaluate Expressions Without Grouping Symbols*

Evaluate the variable expression when x = 2. Use the order of operations.

1. $2x^2 + 5$ **2.** $8 - x^2$ **3.** $6 + 3x^3$ **4.** $20 - 4x^2$

LEFT-TO-RIGHT RULE Some expressions have operations that have the same priority, such as multiplication and division *or* addition and subtraction. The **left-to-right rule** states that when operations have the same priority, you perform them in order from left to right.

EXAMPLE 2 *Use the Left-to-Right Rule*

Evaluate the expression using the left-to-right rule.

a. $24 - 8 - 6 = (24 - 8) - 6$ Work from left to right.

$= 16 - 6$ Subtract 8 from 24.

$= 10$ Subtract 6 from 16.

b. $15 \cdot 2 \div 6 = (15 \cdot 2) \div 6$ Work from left to right.

$= 30 \div 6$ Multiply 15 times 2.

$= 5$ Divide 30 by 6.

c. $16 + 4 \cdot 2 - 3 = 16 + (4 \cdot 2) - 3$ Do multiplication first.

$= 16 + 8 - 3$ Multiply 4 times 2.

$= (16 + 8) - 3$ Work from left to right.

$= 24 - 3$ Add 16 and 8.

$= 21$ Subtract 3 from 24.

A fraction bar can act as a grouping symbol: $(1 + 2) \div (4 - 1) = \dfrac{1 + 2}{4 - 1}$.

EXAMPLE 3 *Expressions with Fraction Bars*

Evaluate the expression. Then simplify the answer.

$$\frac{7 \cdot 4}{8 + 7^2 - 1} = \frac{7 \cdot 4}{8 + 49 - 1}$$ Evaluate power.

$$= \frac{28}{8 + 49 - 1}$$ Simplify the numerator.

$$= \frac{28}{57 - 1}$$ Work from left to right.

$$= \frac{28}{56}$$ Subtract.

$$= \frac{1}{2}$$ Simplify.

Checkpoint ✓ *Use the Order of Operations and Left-to-Right Rule*

Evaluate the variable expression when *x* = 1.

5. $4x^2 + 5 - 3$ **6.** $5 - x^3 - 1$ **7.** $\dfrac{2x}{x^2 - 1 + 5}$

USING A CALCULATOR You need to know if your calculator uses the order of operations or not. If it does not, you must input the operations in the proper order yourself.

EXAMPLE 4 *Use a Calculator*

Enter the following in your calculator. Does the calculator display 6 or 1?

10 [−] 6 [÷] 2 [−] 1 [ENTER]

Solution

a. If your calculator uses the order of operations, it will display 6.

$$10 - 6 \div 2 - 1 = 10 - (6 \div 2) - 1$$
$$= (10 - 3) - 1$$
$$= 6$$

b. If your calculator does *not* use the order of operations and performs the operations as they are entered, it will display 1.

$$[(10 - 6) \div 2] - 1 = (4 \div 2) - 1$$
$$= 2 - 1$$
$$= 1$$

Link to
Sports

BASKETBALL SCORES
A field goal is worth 2 points.
A free throw is worth 1 point.

EXAMPLE 5 *Evaluate a Real-Life Expression*

BASKETBALL SCORES You are playing basketball. You make 8 field goals and 2 free throws. Your friend makes half as many field goals as you and no free throws. How many points ahead of your friend are you?

Solution

$$8 \cdot 2 + 2 \cdot 1 - \frac{8 \cdot 2}{2} = 16 + 2 - \frac{16}{2} \qquad \text{Multiply from left to right.}$$
$$= 16 + 2 - 8 \qquad \text{Divide.}$$
$$= 18 - 8 \qquad \text{Add.}$$
$$= 10 \qquad \text{Subtract.}$$

ANSWER ▶ You are 10 points ahead of your friend.

Checkpoint ✓ *Evaluate a Real-Life Expression*

8. Your friend makes 4 field goals. You make three times as many field goals as your friend plus one field goal. How many points do you have? Explain the order of operations you followed.

9. Your friend makes 6 field goals and 2 free throws. You make twice as many field goals as your friend and half the number of free throws. How many points do you have? Explain the order of operations you followed.

1.3 *Order of Operations* **17**

Guided Practice

Vocabulary Check

1. Place the operations in the order in which you should do them.

    a. Multiply and divide from left to right.

    b. Do operations within grouping symbols.

    c. Add and subtract from left to right.

    d. Evaluate powers.

2. What rule must be applied when evaluating an expression in which the operations have the same priority?

Skill Check

Evaluate the expression.

3. $5 \cdot 6 \cdot 2$ **4.** $16 \div 4 - 2$ **5.** $4 + 9 - 1$ **6.** $2 \cdot 8^2$

7. $15 + 6 \div 3$ **8.** $9 \div 3 \cdot 2$ **9.** $2 \cdot 3^2 + 5$ **10.** $2^3 \cdot 3^2$

Evaluate the variable expression when $x = 3$.

11. $x^2 - 5$ **12.** $x^3 + 5x$ **13.** $x + 3x^4$

14. $\dfrac{27}{x} - 2 + 16$ **15.** $\dfrac{15}{x} + 2^3 - 10$ **16.** $\dfrac{24}{x} \cdot 5$

Practice and Applications

NUMERICAL EXPRESSIONS Evaluate the expression.

17. $13 + 3 \cdot 7$ **18.** $7 + 8 \div 2$ **19.** $2^4 - 5 \cdot 3$

20. $6^2 + 4$ **21.** $4^3 + 9 \cdot 2$ **22.** $3 \cdot 2 + \dfrac{5}{9}$

VARIABLE EXPRESSIONS Evaluate the expression for the given value of the variable.

23. $6 \cdot 2p^2$ when $p = 5$ **24.** $2g \cdot 5$ when $g = 4$

25. $14(n + 1)$ when $n = 2$ **26.** $\dfrac{x}{7} + 16$ when $x = 14$

NUMERICAL EXPRESSIONS Evaluate the expression.

27. $2^3 + 5 - 2$ **28.** $4 \cdot 2 + 15 \div 3$ **29.** $6 \div 3 + 2 \cdot 7$

30. $5 + 8 \cdot 2 - 4$ **31.** $16 + 8 \cdot 2^2$ **32.** $2 \cdot 3^2 - 7$

33. $10 - 3 + (2 + 5)$ **34.** $7 + 18 - (6 - 3)$ **35.** $[(7 \cdot 4) + 3] + 15$

Student Help

▶ **HOMEWORK HELP**
Example 1: Exs. 17–26
Example 2: Exs. 27–35
Example 3: Exs. 36–41
Example 4: Exs. 43–46
Example 5: Exs. 47–53

EXPRESSIONS WITH FRACTION BARS Evaluate the expression. Then simplify the answer.

36. $\dfrac{6 \cdot 4}{4 + 3^2 - 1}$

37. $\dfrac{13 - 4}{18 - 4^2 + 1}$

38. $\dfrac{5^2 \cdot 2}{1 + 6^2 - 12}$

39. $\dfrac{21 + 9}{5^2 + 40 - 5}$

40. $\dfrac{3^3 + 8 - 7}{2 \cdot 7}$

41. $\dfrac{4 \cdot 2^5}{16 - 4^2 + 1}$

42. LOGICAL REASONING Which is correct?

A. $\dfrac{9^2 + 3}{5} = 9^2 + 3 \div 5$

B. $\dfrac{9^2 + 3}{5} = [9^2 + 3] \div 5$

CRITICAL THINKING In Exercises 43–46, two calculators were used to evaluate the expression. Determine which calculator performed the correct order of operations.

43. 15 $\boxed{-}$ 6 $\boxed{\div}$ 3 $\boxed{\times}$ 4 $\boxed{\text{ENTER}}$

Calculator A: 12   Calculator B: 7

44. 15 $\boxed{-}$ 9 $\boxed{\div}$ 3 $\boxed{+}$ 7 $\boxed{\text{ENTER}}$

Calculator A: 19   Calculator B: 9

45. 15 $\boxed{+}$ 10 $\boxed{\div}$ 5 $\boxed{+}$ 4 $\boxed{\text{ENTER}}$

Calculator A: 21   Calculator B: 9

46. 4 $\boxed{\times}$ 3 $\boxed{\div}$ 6 $\boxed{\div}$ 2 $\boxed{\text{ENTER}}$

Calculator A: 9   Calculator B: 15

FOOTBALL UNIFORMS
In Exercises 47 and 48, use the table showing the costs of parts of a football player's uniform.

47. A sporting goods company offers a $2000 discount for orders of 30 or more complete uniforms. Your school orders 35 complete uniforms. Write an expression for the total cost.

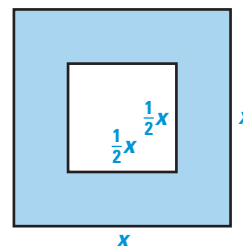**48.** Evaluate the expression you wrote in Exercise 47.

Part of uniform	Jersey and pants	Shoulder pads	Lower body pads	Knee pads	Cleats	Helmet
Cost	$230	$300	$40	$15	$100	$200

49. Write an expression that represents the area of the shaded region. *HINT:* Subtract the area of the inner square from the area of the outer square.

50. If $x = 8$, what is the area of the shaded region?

ADMISSION PRICES In Exercises 51 and 52, use the table below. It shows the admission prices for the California State Fair.

California State Fair Admission Prices	
Age	**Admission price**
General Admission (13–61 years of age)	$7.00
Seniors (62 years and above)	$5.00
Children (5–12 years)	$4.00
Children (4 years and under)	Free

▶ Source: *Sacramento Bee*

51. Write an expression that represents the admission price for a group consisting of 2 adults, 1 senior, and 3 children. The children's ages are 12 years, 10 years, and 18 months.

52. Evaluate the expression you wrote in Exercise 51. Then find the total cost of admission for the group.

53. **CHALLENGE** At a concert you buy a hat for $10.00, a hot dog for $2.75, and nachos for $3.50. There is a 6% sales tax on the hat. Your calculator follows the established order of operations. Write a keystroke sequence for the amount you owe. Then find the amount you owe. *HINT:* 6% = 0.06

Standardized Test Practice

54. **MULTIPLE CHOICE** Evaluate the expression $4^2 - 10 \div 2$.

   Ⓐ 3   Ⓑ 11   Ⓒ 13   Ⓓ 21

55. **MULTIPLE CHOICE** Evaluate the expression $32 - x^2 + 9$ when $x = 2$.

   Ⓕ 19   Ⓖ 21   Ⓗ 37   Ⓙ 39

56. **MULTIPLE CHOICE** Which expression has a value of 12?

   Ⓐ $3 + 3 \times 5 - 2$

   Ⓑ $18 \div 6 \times 3 + 3$

   Ⓒ $7 + 14 \div 7 \times 4$

   Ⓓ $2^2 \cdot 3 - 6 \cdot 2$

57. **MULTIPLE CHOICE** Evaluate the expression $\dfrac{3^2 + 6 - 5}{2 \cdot 5}$.

   Ⓕ 1   Ⓖ 5   Ⓗ 7   Ⓙ 10

Link to State Fairs

ADMISSION PRICES Every year nearly 1,000,000 people attend the California State Fair.

EVALUATING EXPRESSIONS **Evaluate the expression for the given value of the variable.** *(Lesson 1.1)*

58. $(8)(a)$ when $a = 4$ **59.** $\dfrac{24}{x}$ when $x = 3$ **60.** $c + 15$ when $c = 12$

61. $\dfrac{x}{2} \cdot x$ when $x = 18$ **62.** $9t$ when $t = 7$ **63.** $25 - y$ when $y = 14$

WRITING POWERS **Write the expression in exponential form.** *(Lesson 1.2)*

64. twelve squared   **65.** z to the sixth power   **66.** $2b \cdot 2b \cdot 2b$

EXPONENTIAL EXPRESSIONS **Evaluate the expression for the given value of the variable.** *(Lesson 1.2)*

67. $9t^2$ when $t = 3$ **68.** $(7h)^3$ when $h = 1$ **69.** $(6w)^2$ when $w = 5$

FACTORS **Determine whether the number is prime or composite. If it is composite, list all of its factors.** *(Skills Review p. 761)*

70. 15   **71.** 9   **72.** 13   **73.** 38

74. 46   **75.** 50   **76.** 64   **77.** 29

Quiz 1

Evaluate the variable expression when $x = 3$. *(Lessons 1.1, 1.2)*

1. $6x$ **2.** $42 \div x$ **3.** $x + 29$

4. $12 - x$ **5.** $5x - 10$ **6.** $10 + 2x$

7. $x^2 - 3$ **8.** $2x^3$ **9.** $(2x)^3$

Find the distance traveled using $d = rt$. *(Lesson 1.1)*

10. A car travels at an average speed of 50 miles per hour for 4 hours.

11. A plane flies at 500 miles per hour for 4 hours.

12. A marathon runner keeps a steady pace of 10 miles per hour for 2 hours.

Write the expression in exponential form. *(Lesson 1.2)*

13. six cubed   **14.** $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$ **15.** $5y \cdot 5y \cdot 5y$

16. $3 \cdot 3 \cdot 3$ **17.** $2x \cdot 2x \cdot 2x \cdot 2x$ **18.** eight squared

19. **PACKING BOXES** A cubic packing box has dimensions of 4 feet on each edge. What is the volume of the box? *(Lesson 1.2)*

Evaluate the expression. Then simplify the answer. *(Lesson 1.3)*

20. $\dfrac{7 \cdot 2^2}{7 + (2^3 - 1)}$ **21.** $\dfrac{(3^2 - 3)}{2 \cdot 9}$ **22.** $\dfrac{6^2 - 11}{2(17 + 2 \cdot 4)}$